3 1761 05478677 7

MEASUREMENT AND MECHANICS

Presented to

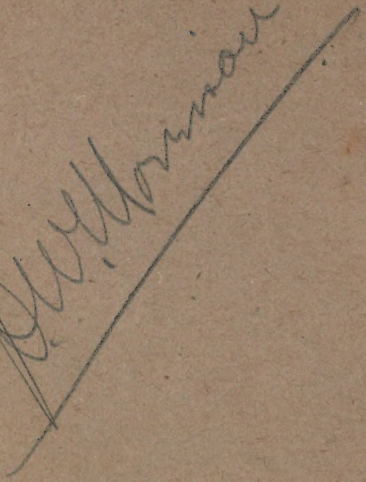
The Library

of the

University of Toronto

by

C. E. Morrison, Esq.



MEASUREMENT

AND

MECHANICS

BY

JOHN SATTERLY, D.Sc., M.A.

345093
30. 12. 37.

LONDON: W. B. CLIVE

University Tutorial Press Ltd.

HIGH ST., NEW OXFORD ST., W.C.

1913

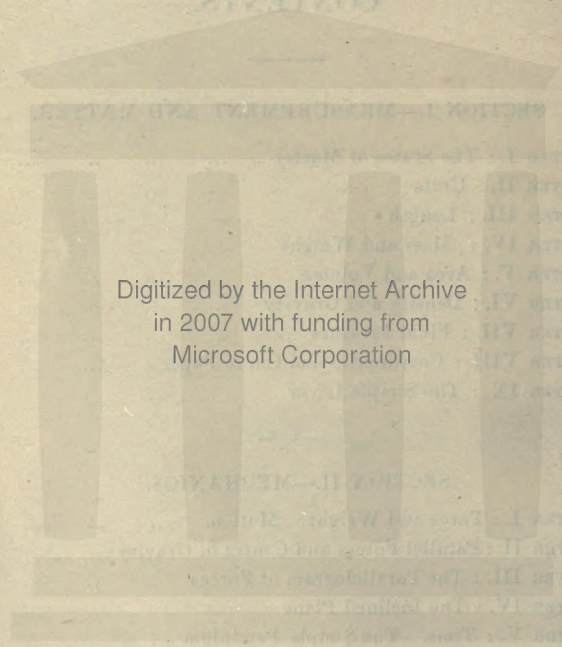
CONTENTS.

SECTION I.—MEASUREMENT AND MATTER.

	PAGE
CHAPTER I. : The States of Matter	1
CHAPTER II. : Units	5
CHAPTER III. : Length	11
CHAPTER IV. : Mass and Weight	23
CHAPTER V. : Area and Volume	31
CHAPTER VI. : Density and Gravity	47
CHAPTER VII. : Fluid Pressure	63
CHAPTER VIII. : Coordinates and Curve Paper	81
CHAPTER IX. : The Simple Lever	85
Chapter X : Force, Weight, Gravity	98
Chapter XI : Molecular physics	103

SECTION II.—MECHANICS.

CHAPTER I. : Force and Weight. Motion.	1
CHAPTER II. : Parallel Forces and Centre of Gravity	13
CHAPTER III. : The Parallelogram of Forces	33
CHAPTER IV. : The Inclined Plane	42
CHAPTER V. : Time. The Simple Pendulum	50
CHAPTER VI. : Couples	57
CHAPTER VII. : Velocity	61
CHAPTER VIII. : Acceleration. Falling Bodies	71
CHAPTER IX. : Force and the Laws of Motion	83
CHAPTER X. : Work, Power, and Energy	98
ANSWERS	119



Digitized by the Internet Archive
in 2007 with funding from
Microsoft Corporation

http://www.archive.org/details/measurementmecha00sattuoft

SECTION I.

MEASUREMENT AND MATTER.

CHAPTER I.

THE STATES OF MATTER.

1. Matter.—All bodies which are known to us consist of substances or materials of varying character. These substances or materials are included under the common name of **matter.**

The **quantity of matter** in a body is termed its **mass.**

2. Volume.—One of the fundamental properties of matter with which we are acquainted is that the same portion of space cannot be filled by different portions of matter at the same time; that is, every body occupies a certain portion of space to the exclusion of any other body. The *measure* of such portion of space is termed the **volume** of the body. Thus the volume of a body is the amount of "room" it takes up.

The volume of a body depends upon its **length, its breadth, its depth,** and its **shape.**

Solids and Fluids.

3. Solids, Liquids, and Gases.—Bodies can be divided into three classes — *solids*, *liquids*, and *gases*. From our everyday experience we get a fairly good idea of the main differences between these. We must now give exact definitions, which may be based on common experience.

We know that a solid body, such as a piece of ice, metal, glass, or wood, always retains the same shape; if put into a

bottle, it does not adapt its shape to that of the bottle. We cannot force a piece of stick into it, nor can we stir it up.

On the other hand, liquids and gases, such as water and air, will flow easily from one vessel into another. If water be poured into a bottle, it adapts itself to the shape of the bottle, and fills the whole of the bottom part. If there is nothing but air in the bottle, there are no empty spaces; the air fills the bottle. Again, water is very easily stirred up with a stick, and air is still more easily stirred, so much so that, when we move about, we experience no perceptible resistance from the air which we displace.

Liquids and gases (*e.g.* water and air) are termed *fluids* on account of their yielding to any force, however small, that tends to change their shape or to produce movements among their parts. They differ in one important respect. If a bottle is half full of water, the water cannot be made to occupy either more or less than half of the bottle. If the bottle is full, we cannot get any more water in by squeezing, nor can we squeeze the water into a smaller space by pushing a cork in or otherwise. On the other hand, any amount of air can be forced into a bottle by pressure, or, again, part of the air in a bottle may be sucked out, and then the remainder will still continue to occupy the *whole* of the bottle. Hence we may distinguish a liquid from a gas by the property that the former cannot, and the latter can, be readily made to occupy a greater or less amount of space. We have thus the following

DEFINITIONS.—**A solid is a body which has definite size and definite shape.** The relative positions of its particles cannot be altered without the application of at least a moderate force. *Examples:* wood, iron, leather.

A liquid is a body which has definite size but no definite shape. It adapts itself to the shape of the containing vessel. Its particles can be separated by the application of a very slight force. *Examples:* water, oil.

A gas is a body which has neither definite size nor definite shape. It tends to increase indefinitely in volume as the pressure confining it within a certain space is removed. It always fills the containing vessel. *Examples:* air, oxygen.

Notice that **gases are distinguished from liquids** by

(i.) Their **compressibility,** in virtue of which they can be compressed into any volume, however small (until they liquefy). by the application of sufficiently great pressure.

(ii.) Their **elasticity,** in virtue of which they expand when the pressure is reduced, so as always to fill the whole volume, however large, of the containing vessel, and exert pressure on its sides.

It is probable that *most* bodies can exist in any one of the three states: solid, liquid, or gaseous. Many we know do so.

Examples.—The liquid, water, when cooled becomes the solid, ice; when heated to 100° C. it becomes the gas, steam.

On the other hand, the gases oxygen and air have been converted into liquids and solids by means of great pressure and low temperature.

4. The Viscous and Plastic States. — A body may exist in such a condition that it is impossible to say that it is solid, liquid, or gas. And when the body can exist either as a solid or liquid or gas, the change from one state to another is frequently gradual. The following experiments illustrate these conditions :—

Exp. 1.—Place on a sheet of glass a drop of water, a small quantity of treacle, and a piece of wax. Slightly tilt the glass. Notice that the water flows down at once, leaving a very *thin* trail. The treacle shows a tendency to *flow;* the wax remains. Incline the glass further. The treacle will flow, leaving a *thick* trail, and the wax will show a tendency to slide.

Now, the trail is due to that portion of the matter, water or treacle, which is in *contact* with the glass; and it is, therefore, the *upper* layers of the matter which *flow over* the lower layers. In other words, *separation* between the upper layers and lower layers occurs in the case of water and of treacle, but not apparently in the case of the wax at ordinary temperature. But there is this difference: the water separates with the greatest ease, the treacle with some difficulty, and the wax not at all.

Definition.—A fluid in which one layer does not easily flow over another is termed a **viscous** fluid, and this property of a fluid is termed **viscosity.**

We may therefore sum up thus:

(*a*) Water *flows* without difficulty, and is a *non-viscous fluid.*

(*b*) Treacle *flows* with difficulty, and is a *viscous* fluid.

(*c*) Wax does *not flow* at ordinary temperatures, and in character approaches very near to a solid.

Exp. 2.—Now gently warm the glass under the wax. After a short time the under layers of wax will melt and spread. Tilt the glass and notice that the upper layers of wax flow over the lower layers. Hence, at higher temperature wax becomes a *viscous fluid*.

It will be found, however, that at intermediate temperatures the wax is neither a solid nor a viscous fluid. It will resemble the treacle in the fact that it is easy to make a dent in either, but the dent in the treacle will be rapidly filled up and disappear, whilst that in the wax will remain for a considerable time, and, it may be, permanently. At these intermediate temperatures wax can be moulded between the fingers.

DEFINITION. — The **plastic** state is a state intermediate between the viscous and the solid state. In this state little force is required to produce change of shape, and such change when produced is to a large degree permanent.

5. Other Properties of Matter. — There are many other qualities of bodies which should be considered. Matter, for example, cannot be destroyed; but the experiments showing this belong rather to chemistry than to mechanics.

Again, bodies of the same size have different weights or masses, *i.e.* some bodies are more dense than others.

For example, we say "Lead is denser than wood, and wood is denser than cork." This property will be described more fully in Chap. VII.

Summary.—Chapter I.

1. The *mass* of a body is the quantity of matter it contains; the *volume* of a body is the space it occupies. (§§ 1, 2.)

2. All bodies may be divided into two classes: *solids* and *fluids*. Fluids may be further divided into *liquids* and *gases*. (§ 3.)

3. Solids and liquids have definite size; but, whereas solids have definite shape, liquids take the shape of the part of the vessel they are in. Gases always fill the containing vessel. (§ 3.)

4. A solid is said to be in the plastic state when it can be easily moulded. A viscous fluid is one in which considerable friction is exerted between its component parts. (§ 4.)

CHAPTER II.

UNITS.

6. Space, Mass, and Time.—In Natural Philosophy we have to deal with three fundamental ideas, namely: **space, mass,** and **time.** It is difficult, or impossible, to give an exact *definition* of either of these three terms, but they are so familiar to us that this is hardly necessary. It is much more important to show how they can be measured; for in Natural Philosophy exact measurements of all the quantities with which we are dealing are of the utmost importance.

7. Units.—In order to measure any one of the properties of a body, such as its length or mass, we fix upon a certain quantity of the same kind and call it our **unit of measurement.** We can then express any quantity of the same kind by specifying

(i.) the **name** of the unit chosen,

(ii.) the **number** of times the quantity contains that unit.

For example, in expressing the age of a child we must first settle upon a certain period of time as our unit. If we choose a year as the unit, the age may be stated as being, say, 2 years, where *year* is the *name of the unit* chosen, and 2 is the *number of times* the age of the child contains that unit.

We are not compelled to take a *year* as the unit. If we had selected a *month*, the age would be 24 *months*; if a *day* had been chosen as the unit, the age would be 730 *days*. But in all cases **the name of the unit must be mentioned** as well as the number of times the unit is contained in that quantity.

The units of space, mass, and time are chosen as fundamental in preference to other units because the units of all other physical quantities can be derived from them. Also our earliest conceptions of dimension are those of space, mass, and time, and it is therefore natural that we should derive the units of other physical quantities from these.

The numerical value of a certain quantity is generally deduced by calculation from the data obtained by measuring

quantities directly or indirectly associated with it. For instance, the volume of a body may be measured directly by the use of a graduated vessel, indirectly by finding its dimensions in certain directions, or its mass and density, or by Archimedes' Principle.

8. Two Systems of Units.—As a result of the freedom of choice in the magnitude of the fundamental units, the systems of all nations are not the same. We shall have to deal in this work with two systems: the **English** and the **French or Metric** systems. Care must be taken to keep these perfectly distinct. A table connecting the two series of units is given in § 13.

9. The English or foot-pound-second system.—

(*a*) The **unit of length or distance** is the **foot.** This is one-third of a **yard,** which is defined by Act of Parliament to be the distance between the centres of two marks in a certain bronze bar kept in the Board of Trade offices at London.

As the length of a body changes with the temperature the measurement is to be taken when the bar is at a temperature of 62° Fahrenheit.

The area of a body is the measure of its surface.

The unit of area is the area of a square the length of any side of which is 1 foot, *i.e.* the **unit of area** is **1 square foot.**

The unit of volume is the volume of a cube the length of any side of which is 1 foot, *i.e.* the **unit of volume** is **1 cubic foot.**

(*b*) The **unit of mass** is the **pound.** This is defined as the mass of a certain piece of platinum kept at the Board of Trade offices.*

(*c*) The **unit of time** is the **mean solar second.** This is the $\frac{1}{24\times 60\times 60}$ th part of the *average* length of the solar day, and is very nearly the time taken at Greenwich by a pendulum 39·139 inches long to make one beat.

For brevity, the words foot, pound, and second are commonly written **ft., lb.,** and **sec.,** and the system is called the **F.P.S.** system.

* In the *Weights and Measures Act* the pound is defined as the legal standard of *weight*, because the term "weight" is commonly used to denote "mass," and masses are commonly compared by "weighing" them. (§ 23.)

10. The Metric or centimetre - gramme - second system.—

(*a*) The **unit of length or distance** is the **centimetre (cm.)**. This is one hundredth part of the **metre,** which is defined by French law as the length of a certain rod of platinum at a temperature of 0° C., which is kept in the Archives at Paris.*

The **unit of volume** is the **cubic centimetre (cub. cm.** or **c.c.).** Very frequently volumes are expressed in terms of the **litre,** which is nearly equal to the *cubic decimetre.*

Thus the litre = 1,000 cubic centimetres very nearly.

(*b*) The **unit of mass** is the **gramme (gm.).** This is the one-thousandth part of a kilogramme, which is the mass of a certain lump of platinum kept at Paris. It is very nearly equal to the mass of a cubic centimetre of pure water at a temperature of 4° C.†

(*c*) The **unit of time** is the same as in the English system, *i.e.* it is the **mean solar second.**

The student will be familiar with the multiples and submultiples of the yard, pound, and second.

The multiples and submultiples of the metre and the gramme are shown below. Those in dark type should be remembered.

A metre = **10 decimetres,**

= **100 centimetres** (cm.),

= 1000 **millimetres** (mm.).

---

* At the introduction of the Metric System the metre was defined as the ten-millionth part of the length of the quadrant of the Earth's circumference measured from the North Pole to the Equator, and the gramme as the mass of a cubic centimetre of water at 4° C. Since then the quadrant of the Earth has been re-measured and found to be not quite 10,000,000 metres, and a cubic centimetre of water has been re-weighed and found to be not quite 1 gramme. But the original metre and gramme have been retained; hence the metre is now defined as the length of a certain rod and the kilogramme as the mass of a certain lump of platinum.

† It was necessary to select a certain temperature when the gramme was defined, since the mass of a given volume of water varies with the temperature of the water.

10 metres = 1 **deca**metre.
100 „ = 1 hectometre.
1000 „ = 1 **kilo**metre.

A gramme = 10 **deci**grammes,
= 100 centigrammes,
= 1000 **milligrammes.**
10 grammes = 1 **deca**gramme.
100 „ = 1 hectogramme.
1000 „ = 1 **kilo**gramme.

11. The **centimetre-gramme-second system** is briefly described as the **C.G.S.** system. Its **advantages** are as follows:—

(i.) To convert or "reduce" a unit to its multiples or submultiples, we have only to multiply or divide by some power of 10, and this can be effected *at sight* by moving the decimal point, or adding or taking off cyphers.

(ii.) The units of length, volume, and mass bear a simple relation to one another. Thus we can write down at once the volume of a body of water in cubic centimetres if we know its mass in grammes, and *vice versa*.

On account of these advantages the C.G.S. system is used largely in some countries for purposes of internal and external trade. It is also used almost wholly in all countries for scientific measurements. In England and America, however, it has failed to oust the F.P.S. system from the world of engineering.

12. Diagram of the Metric System. — The opposite diagram (Fig. 1) represents a cube whose side is 1 decimetre, the lengths on its front face being drawn to scale. This large cube would hold a kilogramme of water at 4° C., while the small cube at the left-hand top corner would hold a gramme of water at the same temperature.

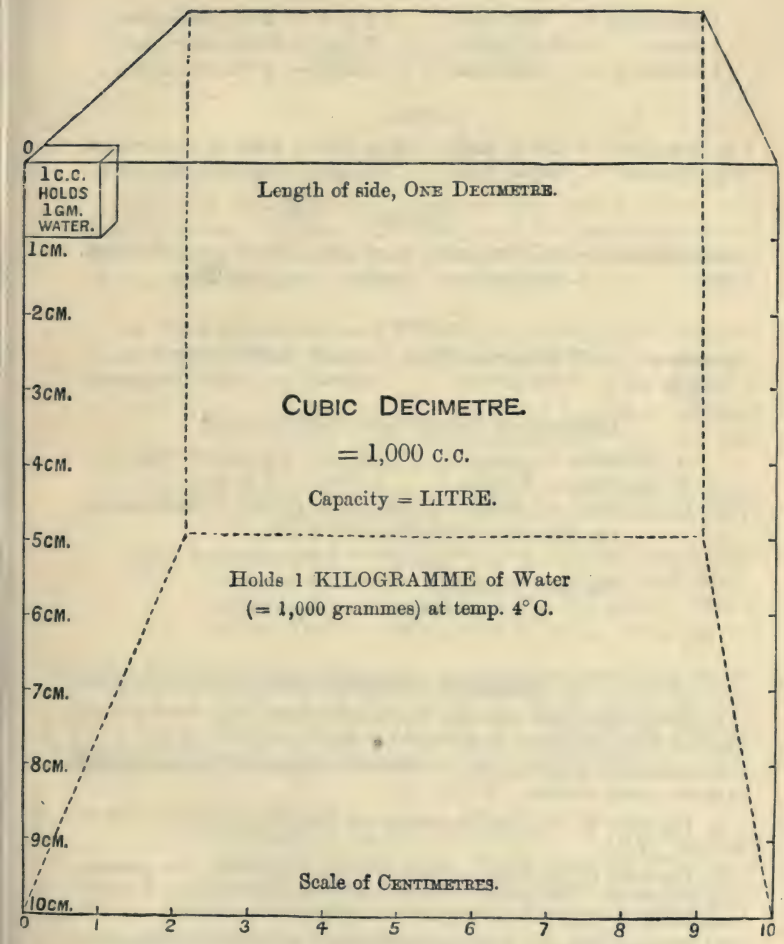

0
1 C.C. HOLDS 1 GM. WATER.
Length of side, One Decimetre.
1 CM.
2 CM.
3 CM.
CUBIC DECIMETRE.
= 1,000 c.c.
Capacity = LITRE.
4 CM.
5 CM.
Holds 1 KILOGRAMME of Water
(= 1,000 grammes) at temp. 4° C.
6 CM.
7 CM.
8 CM.
9 CM.
Scale of Centimetres.
10 CM.
0 1 2 3 4 5 6 7 8 9 10

Fig. 1.

13. Tables.—

LENGTH.

1 centimetre =	·3937 inch.	1 inch =	2·54 centimetres.
1 metre =	39·37 inches.	1 foot =	30·48 centimetres.
1 kilometre =	·6213 mile.	1 mile =	1·609 kilometres.

AREA.

1 sq. centimetre = ·155 sq. inch.	1 sq. inch = 6·452 sq. centimetres.
1 sq. kilometre = ·386 sq. mile.	1 sq. mile = 2·590 sq. kilometres.

VOLUME.

1 cub. centimetre = ·0610 cub. inch.	1 cub. inch = 16·39 cub. centimetres.
1 litre = ·2205 gallons.	1 gallon = 4·536 litres.

MASS.

1 gramme = 15·432 grains (Troy).	1 ounce = 28·35 grammes.
1 kilogramme = 2·205 pounds.	1 pound = ·4536 kilogramme.

APPROXIMATE VALUES (useful to remember).

1 millimetre = $\frac{1}{25}$ inch.	1 foot = 30½ centimetres.
8 kilometres = 5 miles.	1 litre = 1¾ pints.

1,000 kilogrammes or 1 *tonne* = 2,**204** pounds (a ton = 2,**240** pounds).

The diameter of a halfpenny = 1 inch.
,, ,, penny = 3 centimetres.
The mass of a penny = $\frac{1}{3}$ ounce.
,, ,, sovereign = 8 grammes.

Summary.—Chapter II.

1. *Space*, *mass*, and *time* are three fundamental ideas, and all other physical magnitudes can be expressed in terms of these. (§ 6.)

2. *Quantities* of space, mass, or time are expressed in terms of a *unit* of space, mass, or time. (§ 7.)

3. The *units* in the English system are the foot, the pound, and the second. (§ 9.)

4. The units in the metric system are the centimetre, the gramme, and the second. The gramme is very nearly equal to the mass of a cubic centimetre of water at the temperature at which it is densest. (§ 10.)

5. The metric system has many advantages over the English system. The chief point in its favour is that it greatly simplifies calculations. (§ 11.)

CHAPTER III.

LENGTH.

THIS and the following chapters deal more fully with Length, Area, Volume, and Mass, and give some account of the means by which they are measured, and a description of the instruments used.

14. The Measurement of Length.—Graduated straight rods of wood or flat bars of steel are used for this purpose. In common use are :—

(1) The ordinary foot-rule, made of boxwood, divided into inches and subdivisions, viz. eighths, sixteenths, twelfths, and tenths.

(2) The metre stick or scale, made of boxwood, divided into centimetres and millimetres.

(3) A scale, just over a foot long, divided into inches and subdivisions along one edge, and centimetres and millimetres along the other. Fig. 2 shows such a scale made of steel, which proves

Fig. 2.

very useful in the laboratory. A similar scale made of boxwood and bevelled along both edges is very useful for graphical work, *i.e.* for work involving accurate drawing and measurement.

(4) A flexible steel tape, seven or eight feet long, graduated on one face in feet and inches, and on the other in centimetres and millimetres.

Exp. 3.—Place an inch-rule graduated to tenths or eighths and a centimetre-rule graduated to millimetres edge to edge (Fig. 3), and find how many centimetres are equal to a foot. If the scales are accurate, you should obtain the number 30·5.

Fig. 3.

Do not read from the ends of the scales, as the end divisions are usually short through being worn. It is better to place the 1 centimetre mark opposite to the 1 inch mark. Read off at the places where graduations on one scale are exactly opposite to graduations on the other scale. Repeat several times, starting at different graduations, and work out from each reading the value of 1 foot in centimetres.

The results will not exactly agree, because it is impossible at any one reading to avoid making an error. As in the long run as many errors are likely to be made one way as the other, the average or mean value of all your readings should be very near the truth.

Never be content with a single reading. Take four or five if time permits.

Exp. 4.—Measure the distance between two parallel straight lines. If the distance is not an exact number of scale divisions, the length of the bit over should be estimated by the eye.

In Fig. 4 observe (1) that there would be inaccuracy at *A* because the end of the scale is not "square." Also observe (2) that, owing to the thickness of the rule, the reading of *B* would be judged differently when the position of the eye is changed. In the figure it is 1·63 when viewed along *mB*, 1·58 along *nB*, 1·48 along *pB*. This possible error

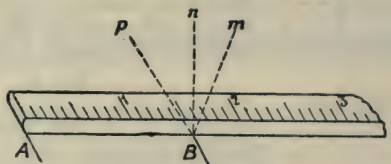


Fig. 4.

is said to be due to **parallax**, and, with the rule in this position, it will always occur unless the eye is placed along a line (such as nB) at right angles to the paper through the point whose position is being measured. To get an accurate reading, stand the rule on its edge (Fig. 5), so that its division marks actually touch the paper.

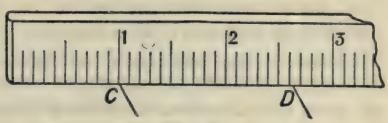


Fig. 5.

In Fig. 5 the position of C is 1·00, of D, 2·64 ;

$$\therefore \quad CD = 2{\cdot}64 - 1{\cdot}00 = 1{\cdot}64 \text{ inches.}$$

Take the mean of several readings. Be careful in expressing the answer to state the unit of measurement. Thus, to say the distance is "25" is to give a meaningless answer. An answer such as "25 centimetres" is, however, definite and intelligible.

Exp. 5.—*Measure the width of a neck, or diameter of a cylinder* (Fig. 6).

Rest a scale and set-squares on books. Place the cylinder or neck (of a bottle) between the set-squares. Press together so that one side of each set-square is in contact with the edge of the scale, and the second side of each set-square touches the cylinder. Then the distance between the set-squares equals the width of neck or diameter of cylinder.

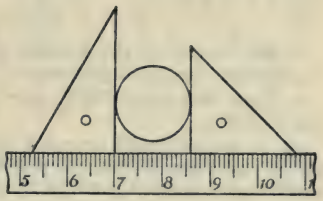


Fig. 6.

The diameter of a tube, or cylinder, should be measured at different positions along its surface in order to test the uniformity of its figure. The diameter of a sphere, or the height of a cone, may be measured in the same way. Instead of the set-squares, two *rectangular* blocks may be used.

This illustrates the principle of the *sliding calipers* (§ 18).

EXERCISES I.

Practical.

1. Using the rules graduated in inches and centimetres, draw lines to represent to the same scale:—

(i.) an inch and a centimetre; (iii.) a yard and a metre;
(ii.) an inch and a decimetre; (iv.) a mile and a kilometre.

2. Measure the distance in millimetres from the top ruled line to the bottom ruled line of your exercise book.

3. Find in millimetres the shortest distance between any two adjacent lines on the page of your exercise book,

(*a*) by directly measuring it with a millimetre scale, taking the mean value of several readings;

(*b*) by calculation from your answer in Ex. 2. On which answer would you place most reliance? Why?

4. Find the thickness of a page of this book.

5. Without using the scale, draw a straight line that you think to be 1 decimetre long, and another that you think to be 5 inches long. Then measure them with your scale to see how correct your guesses have been.

6. Test the accuracy of your straight-edge. Draw a line on paper with it. Then turn it over and draw another, so that the ends of the two lines are together. If there is any space between the two lines thus drawn, they are not straight, and, consequently, your straight-edge is not accurate.

7. Measure in inches and centimetres the sides of your 60° and 45° set-squares. Show that, when measured in either unit, the sides of the former are in the ratio 1 : 1·73 : 2, and those of the latter in the ratio 1 : 1 : 1·41.

Calculations.

8. The length of the seconds pendulum in London is 39·139 ins. Express this length in centimetres.

9. Mont Blanc is 15,732 ft. above the sea level. Express this height in kilometres.

10. It was originally supposed that the metre was one ten-millionth part of the distance from the pole to the equator. If this were so, what would be the circumference of the Earth in miles?

11. The height of a barometer is stated to be 760 mms. Express this height in inches.

12. A body is dropped from a tower. At the end of 1 second its velocity is 32·2 ft. per second. Find its velocity in centimetres per second.

13. The distance from Dover to Calais is 21 miles. Express this length in kilometres.

14. The wave length of sodium light is 5900×10^{-8} cms. Express the wave length as a fraction of an inch.

Instruments used in Exact Measurements.

15. Compasses and Calipers.—A short length, to which for any reason the scale itself cannot be applied, is conveniently measured by compasses. The legs are opened out and adjusted until the distance between their ends equals that under measurement. The distance is then carefully transferred to a scale and read off. To make sure that the relative positions of the legs of the compasses have not been altered in this operation, the point of one leg is again placed at an end of the length to be measured; the point of the second leg should again just reach the other end of the length.

In compasses the adjustment is made by trial, the distance between the legs being slightly but indefinitely altered by hand. In **hair-dividers** (Fig. 7) and **bow compasses** (Fig. 8) the adjustment is quickly and accurately performed by slightly turning the nut at the side.

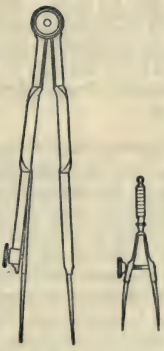


Fig. 7. Fig. 8. Fig. 9.

Sometimes it is not easy to apply a rule to measure the diameter of a cylinder or the bore of a pipe. In such cases **calipers** (Fig. 9) prove very useful. The "inside" end measures internal, and the "outside" external, dimensions. The opening between the tips is adjusted until they touch the surfaces the distance between which is to be measured. For instance, the diameter of a cylinder or sphere is obtained by adjusting the "outside" gap until the body is very slightly pinched. Finally the distance between the ends is measured by a centimetre or inch scale.

When one end of the length to be measured falls between two adjacent graduation marks, the fraction of the division may be approximately guessed to tenths by means of the eye. To get a more accurate reading, a complex scale called a diagonal scale, or an instrument called a vernier, must be used.

16. The **Diagonal Scale** (Fig. 10) is a scale by means of which, by the application of the Principle of Proportion, we are enabled to read to very small distances without marking these distances on the scale itself.

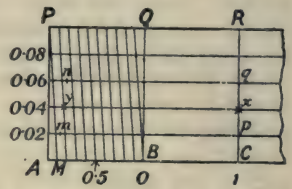


Fig. 10.

The points A, B, C are, say, 1 in. apart. The equal lines AP, BQ, CR, &c., are parallels drawn as a rule perpendicular to AC, and of any convenient length. AB is divided into, say, tenths; AP into, say, fifths. The first division M of AB is joined to P; through the others are drawn parallels to MP. Through each division of AP are drawn parallels to AC. The *least count* or *smallest difference* indicated by the scale is $\frac{1}{5}$ of $\frac{1}{10}$ (or $\frac{1}{50}$) inch. The scale is numbered as shown. When a given length is being measured, the whole number of inches in it will be obtained from the part to the right of BQ; the fraction of the inch will be obtained from the part to the left of BQ. Note that the divisions of AB are numbered **from right to left**: thus B is marked 0, and M is marked 0·9.

To use the diagonal scale.—Suppose it is required to find the length of a given line. Take this length up in the compasses (§ 15), and put the point of one leg of the compass at A. Suppose that the other point lies on AC, between the unit divisions 0, 1. This shows the required length lies between 1 and 2 inches. One compass point is placed on division 1, the other point will lie on BA, say, between divisions 0·8 and 0·9. Thus the base line, AB, shows that the length is between 1·8″ and 1·9″. Now keeping the one compass point on the line CR, through division 1, place it in turn at the points of intersection of this line with the several parallels to the base line AB.

Observe the position of the other compass point on the same parallel: thus, when one point is at p, suppose the other is at m; when at q, then at n; at x, then at y. In the first and second positions it is between diagonals, in the last it is at the intersection of the diagonal through 0·8 and the horizontal through 0·04. Hence the required length is $1 + \cdot 8 + \cdot 04$ or 1·84 in.

This scale is usually provided on the ordinary rectangular form of protractor, and, used by careful hands in conjunction with a good pair of dividers, it affords a good method of measurement.

17. The **Vernier** is a device for readily estimating the fractions of the graduations of a measuring scale; its use avoids the necessity for minute subdivision.

Exp. 6.—*Construct a scale and vernier as follows* (see Fig. 11):—

(1) *Principal Scale.*—On a large piece of paper draw a straight line, and on it mark off by dividers or ruler equal parts—say about $\frac{1}{2}$ inch or 1 centimetre long. Number from left to right: 0, 1, 2, 3, &c.

(2) *Vernier.*—On another strip of paper draw a length equal (say) to 9 divisions of the principal scale. Divide this into 10 equal parts. Mark the left boundary line with an arrow, and number from left to right: 0, 1, 2, 3, 4, 5, ..., 9, 10. Then each division of the vernier is equal to $\frac{9}{10}$ths of a division of the principal scale, and the difference between the two is $\frac{1}{10}$th of the same.

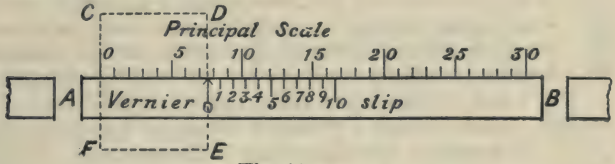


Fig. 11.

(3) Cut out the vernier along the line of graduation, obtaining a strip about $\frac{1}{2}$ inch wide, and arrange it by means of slits at *A* and *B* to slide alongside the principal scale. Put the vernier zero, or mark 0, in line with 10 of scale: vernier division 10 will then coincide with 19 of scale. Now move the vernier until its line 1 coincides with 11 of the scale: the zero of the vernier will then have moved through a distance ($\frac{1}{10}$th in this case) equal the difference between a vernier and scale division. Next bring vernier mark 2 up to 12 of scale: the movement is equal to the previous step, and the total displacement of the zero from its first position is $\frac{1}{10}+\frac{1}{10}$ or $\frac{2}{10}$. Bring, in turn, the succeeding marks of vernier into coincidence with 13, 14, &c., respectively, of the principal scale, and verify the numbers below:

Vernier mark coincident with a scale mark	0, 1, 2, 3, 4, 5, &c.
Scale mark coincident with a vernier mark	10, 11, 12, 13, 14, 15, &c.
Displacement in scale divisions of zero of vernier, from its first position	0, $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, $\frac{5}{10}$, &c.

Exp. 7.—*Measure with the scale and vernier*, constructed as above the length of a pencil, length and breadth of an envelope or card diameter of a penny, &c.

To do this, place (say) the envelope on the paper, as shown by the dotted rectangle *CDEF* (Fig. 11). The left edge must reach exactly to the zero of the principal scale. Adjust the vernier slip until the arrow is underneath the right edge of the envelope. Now, keeping the slip fixed, remove the envelope and study the vernier. The vernier mark lies between the 7th and 8th divisions of the principal scale. Therefore the width of the envelope is between 7 and 8 scale divisions.

To find the exact fraction of the width over 7 divisions, look along the vernier and find where a vernier graduation mark and a scale graduation mark coincide (or most nearly coincide). In the figure it occurs at the 6th mark of the vernier. The distance between the vernier arrow and the 7th mark of the scale is $\frac{6}{10}$ of a scale division. The width of the envelope is therefore

7 scale divisions + $\frac{6}{10}$ scale divisions = 7·6 scale divisions.

Now measure the other objects in the same way. The results will be in terms of the divisions of the principal scale. Find the value of these in inches and centimetres by comparison with a standard measure. Then express the above lengths in inches and centimetres.

Verniers will be found on nearly all instruments used for exact measurements. The most common are those forms which have, as above, 10 vernier divisions equal to 9 scale divisions, and so read to $\frac{1}{10}$th of the scale division.

18. Vernier Calipers (Fig. 12).—The principal scale is graduated on a steel strip, and the vernier on a frame that slides along the strip. A steel jaw projects at right angles to the strip and is fixed at one end of it. Another steel jaw, also at right angles to the strip, forms one end of the slider. The construction is such that the two jaws are in contact when the zero of the vernier, *Z*, coincides with the zero of the principal scale. Hence the scale reading

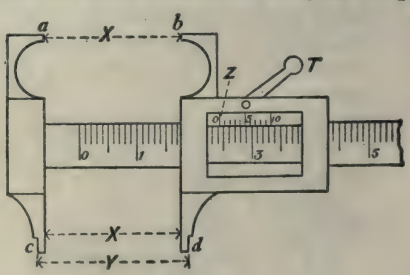


Fig. 12.

at any time measures the gap between the jaws. In Fig. 12 the distance between the tips, *a*, *b*, is equal to that between the jaws, *X*. These, *a*, *b*, can be introduced into tubes, and measure thicknesses that cannot be reached by the jaws, *c*, *d* (Fig. 14). The distance, *Y*, between the outside edges of the projections, *c*, *d*, is 2 millimetres (usually) greater than that between the jaws. These serve to measure the internal diameter of tubes, &c. (Fig. 13). The catch, *T*, clamps the slider to the strip when required. *Be sure that the slider is free before trying to move it.*

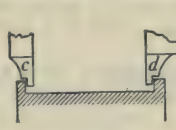


Fig. 13.

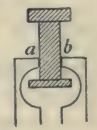


Fig. 14.

The reading in Fig. 12 is 2·43 centimetres.

19. The Divided Circle is another device employed in the accurate measurement of small distances. Examples of this are met in the *Micrometer Screw*, or *Screw Gauge* (Fig. 15), and the *Spherometer* (Fig. 16). In these instruments a screw carrying a circular scale, divided into (say) 100 equal parts, moves along a fixed arm graduated in (say) millimetres. As the screw turns once round, it moves through 1 millimetre along the arm. Now, by means of the circular scale, the amount of turn can be read off correct to the $\frac{1}{100}$th part, and so the distance the screw moves can be estimated to $\frac{1}{100}$ millimetre.

The screw gauge is chiefly used for measuring the thickness of wires and rods, and the spherometer for measuring the thickness of thin plates and the curvature of surfaces.

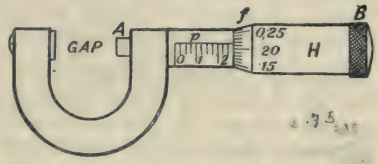


Fig. 15.

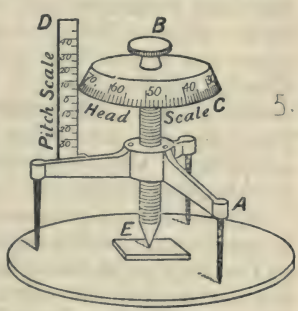


Fig. 16.

EXERCISES II.

Practical.

1. Find the internal diameter of a glass or metal tube (*a*) by means of a millimetre rule; (*b*) by means of a metal wedge or cone; (*c*) by means of calipers or compasses.

2. Measure the diameter of the given piece of wire (*a*) by means of a micrometer screw; (*b*) by coiling the wire tightly round a lead pencil, and measuring the breadth of 10 or 12 turns.

3. Measure the thickness of a microscope coverslip by means of a spherometer.

4. Draw a square of 3 inches side. Find, by using dividers and a diagonal scale, the length of a diagonal.

20. Measurement of Curved Lines.—A straight-edge cannot be directly applied to the measurement of the length of a curved line. Instead of this we resort to various devices.

METHOD I.—*By spring bow compasses* or *dividers.*—Open the points a short distance (about $\frac{1}{4}$ inch, or *less* if the curvatures are sharp) and step along the curve from end to end.* Count the number of steps. Take, say, 20 along a measuring scale, and note the distance traversed. Then (*length of curve*) ÷ (*length on scale*) = (*number of steps on curve*) ÷ (*number of steps on scale*).

METHOD II.—*By thread.*—Place an end of the thread at the left extremity of the line,* and hold it there by pressing it with the nail of the left-hand forefinger. Adjust a short length ($\frac{1}{2}$ inch, say) of the thread *along the curve* and fix by the nail of the right-hand forefinger. Place the nail of the left-hand forefinger close to the right and press the thread by it. Then adjust another short length of thread along the curve and repeat the operations until the end is reached. Finally, cut the thread at the end of the curve and stretch it over a scale. Its length will equal that of the curve. A *paper slip* can sometimes be substituted for the thread with advantage. Use it in conjunction with a sharp pencil or pricker, and, finally, lay it along a scale.

METHOD III.—*By means of a disc with a milled edge.*—Mark a certain point on the edge of the disc, then set it vertically so that this point lies on one end of the curve and roll along the curve until the other end is reached. Then transfer it to the ruler and again roll it until it has made just the same number of revolutions as before. The difference between the scale readings gives the length of the curve. The wheel-measure or Opisometer (Fig. 17) is an instrument made specially for this purpose.

Fig. 17.

* If the curve is *closed*, *e.g.* a circle or ellipse start from a mark on it, and continue round until the mark is again reached.

21. Length of the Circumference of a Circle.—In the particular case of the circle the ratio of the circumference to the diameter is the same for all circles, and is denoted by π. π is an incommensurable number, *i.e.* one that cannot be expressed exactly in figures. Its value — approximately 3·1416—can be calculated mathematically. In practice the less correct values $\frac{22}{7}$ and $\frac{355}{113}$ are often used.

Exp. 8.—*Find the circumference of a disc by rolling it along a scale. Deduce the value of* π.

A large coin or canister lid may be used. Make a mark (by gummed paper) on the edge of the disc, rest the edge on a scale with the mark against a unit division, then roll the disc (without slipping) one revolution along the scale: note the division reached by the mark on the disc. Deduce the circumference of the disc. Repeat several times. Calculate the mean value. Measure the diameter. Calculate *circumference ÷ diameter*. The result is an experimental value of π.

Exp. 9.—Draw a straight line freehand, and at least 8 inches long, on paper. Test its straightness by tracing it on tracing paper, reversing the paper, and placing it over the original. The degree to which the two lines coincide is a measure of the accuracy with which your hand works with your eye.

Summary.—Chapter III.

1. Lengths are measured by scales of wood or metal graduated in inches and subdivisions or centimetres and subdivisions. (§ 14.)

2. For more accurate work the spring bow, calipers, diagonal scale, vernier, screw gauge, and spherometer are used. (§§ 15–19.)

3. If n divisions of the vernier scale equal $(n-1)$ divisions of the principal scale, the vernier enables readings to be taken to $\frac{1}{n}$th of a division of the principal scale. (§ 17.)

4. The screw gauge and spherometer work on the principle of the divided circle. (§ 19.)

5. Curved lines may be measured by (1) stepping with dividers; (2) stepping with thread or slip of paper; (3) use of a disc with milled edge. (§ 20.)

6. For all circles the ratio $\frac{\text{circumference}}{\text{diameter}}$ is constant and denoted by π.

$$\pi = 3{\cdot}14159\ldots.$$

$\frac{22}{7}$ is the simplest approximation to π, but $\frac{355}{113}$ is much nearer. (§ 21.)

EXERCISES III.

PRACTICAL.

1. Draw a circle, either 5 inches or 10 centimetres in diameter, the larger the better. Measure the lengths of the circumference by Methods I., II., III. § 20. Find the mean of the three results. Divide this by the length of the diameter, and so deduce a value of π.

2. Find the circumference of the given cylinder (a round tin or anything turned on a lathe will do)—

(*a*) by rolling it along a scale;

(*b*) by wrapping a paper slip round it perpendicular to the axis of the cylinder, pricking through at the overlap, unrolling, and measuring the distance between the pinholes;

(*c*) by winding a fine piece of string around it, and cutting the string off after it has made an exact number of turns, say 10. Unwind, measure the length, and divide by 10.

3. Find the diameter of the same cylinder—

(*a*) by direct measurement, the diameter being the longest chord of the base (the base must be cut perpendicular to the axis;

(*b*) by placing it between two parallel well-cut wooden blocks, and then measuring the shortest distance between the faces of the blocks;

(*c*) by means of calipers.

From the mean of the results of Exx. 2 and 3 calculate π.

4. Find from a map the distances between places (i.) direct, (ii.) by road, (iii.) by rail. Use the methods of **Exp. 4** for straight lines and of § 20 for curved. Refer the lengths to the scale of the map.

CHAPTER IV.

MASS AND WEIGHT.

22. The **Mass** of a body has already been defined (§ 1) as the "quantity of matter" in that body, and we have also stated that it is measured in grammes, pounds, &c. (§§ 9, 10). This is the only simple definition available, but it is a very unsatisfactory one when referring to bodies of different natures. A pound of lead and a pound of sugar are said to have the same mass; but we have no right to say that these contain the same quantity of matter, for they are entirely different kinds of matter.

Two leading properties of mass which must be considered as axioms are:—

(i.) *The masses of two bodies composed of the same material under the same conditions* are in proportion to their volumes.*

(ii.) *The mass of a body is always the same, and is not altered by changing its form or volume.*†

In daily life the word "mass" is not often used, its place is taken by the term "weight," and it would appear that mass and weight are identical; this, however, is not true. There is a very real distinction between mass and weight, for **weight is a force.**

Definition.—The **weight** of a body on the surface of the Earth is the force with which the Earth attracts it.

If a body could be weighed with a spring balance (§ 30), first on the Earth and then on the Moon, it would be found to pull the spring out less, *i.e.* to weigh less, in the second case,

* For example, the bodies should be at the same temperature (Chapters XVII. and XVIII.).

† *E.g.* by altering the temperature of the body.

for the Moon is much smaller than the Earth and does not attract bodies so strongly. Thus the mass of a body would *not* be changed by taking it from the Earth to the Moon; but its *weight* would be changed.

It can be easily proved that the weights of bodies of the *same* material (at the same place) are proportional to their volumes, and therefore to their masses—see above.

It is not, however, easy to prove that for bodies of *different* material the weights are proportional to the masses; nevertheless, for the remainder of this book, it will be assumed that all bodies which have the same weights at the same place also have equal masses. The student will find that, in Physics, as a rule, no ambiguity will be caused by following the ordinary custom and using the word "weight" where strict scientific usage would require the term "mass." It is only in the science of Dynamics (see Chapters IX. and XV.) that the correct use of these terms is essential.

The Measurement of Mass or Weight.

23. The Balance: Weighing.—The balance (Fig. 18) is used for the comparison of weights. For convenience and accuracy it is made with arms of equal lengths and scale-pans of equal weight. The operation of comparing weights is called *weighing*, and a body whose weight is required is usually compared with certain bodies of known weight, *e.g.* the members of a *set of weights*.

In weighing a body it is placed in the left-hand pan and counterpoised by means of these standard masses placed in the other pan.

By the application of the "Principles of Levers"* we know that since the arms are equal the force pulling one end of the lever of the balance downwards is equal to the force pulling the other end down. Now the scale-pans are equal in mass, hence their downward forces are equal, therefore the remaining forces, viz., the weight of the body in the left pan and the weight of the standard masses placed in the right pan are equal. But mass is proportional to weight. Therefore, the mass of the body is equal to the sum of the standard masses which just counterpoise it.

Other weighing machines are the *steelyard*—a balance with unequal arms (§ 29)—and the *spring balance* (§ 30).

* See Chapter XII.

24. Specification of the Balance (Fig. 18). — The following is a useful form for elementary work. The *beam*, *AB*, has a central *knife-edge* or fulcrum, *C*, of agate or steel. This rests on a flat surface of hard material fixed to the top of a support, *Q*. It is midway between, and about 5 or 6 inches from, *two* other *knife-edges*, at *A* and *B*, close to the ends of the beam. The edges of the latter point upwards and support the stirrups, *D*, from which the *scale-pans*, *S*, are hung. The *masses to be compared* are placed in these pans. A *pointer* or *index* is fixed to the beam. This, when the beam is swinging, moves in front of a short scale of equal parts (unit unimportant) fixed at the foot of the pillar. Thus the index marks the position of the beam, which is practically horizontal when the end of the pointer is in front of the middle line of the scale. The *adjusting nuts*, *n* (sometimes there is one only), consist of a screw stem upon which a nut travels. Turning either nut so that it moves towards the right or left displaces the pointer in the opposite way owing

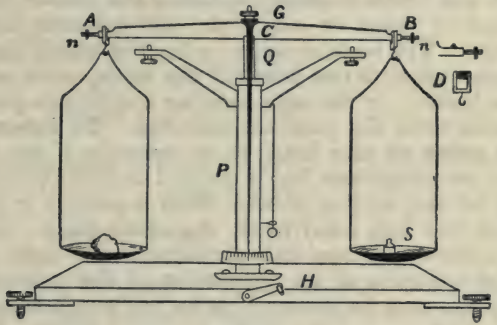


Fig. 18.

to the change in the equilibrium position of the beam. For instance if, when the beam is free, the end of the pointer is opposite division 6 on the left, then it may be brought nearer the middle mark, that is *displaced to the right* by moving *either nut to the left*.

The Arrestment.—The central support, *Q*, can be moved slightly up and down within the pillar, *P*, by the handle, *H*. When the handle is over to the left the support is in its

lowest position, (i.) the beam is lifted from it by the rods and frame springing from and fixed to the top end of the pillar,* (ii.) the scale-pans rest on the base-board. The beam is now *arrested*, or the balance is *out of action*. The arrangement saves wear and tear of the knife-edges. Turning the handle *completely over to the right* (as in Fig. 18) raises the support, and lifts the beam and pans. The beam is now *free* and the balance is *in action*.

The pillar is fixed upright to a base-board, and is sometimes provided with a plummet and the base-board with levelling screws. The pillar may then be adjusted into a vertical position.

25. The Balance in practice.—The balance is in working *adjustment* if, when both pans are unloaded, and the beam free, the end of the pointer moves to and fro in front of the short scale.

The to and fro or vibratory motion of the pointer gradually diminishes in range or *amplitude*, and presently the movement stops. The scale division in front of which the end of the pointer comes to rest is called the *resting point* when the pans are loaded, or the *zero point* or *equilibrium position* of the balance when the pans are unloaded. **To determine a resting point at any load** it is convenient not to wait until the swinging stops, but to proceed as follows:—Observe the end of the moving pointer from a position in front of, and about two feet away from, the short scale. Note the *turning points* or the two scale divisions on the right and left that mark the ends of the movement of the pointer. The resting point may be assumed to be midway between these divisions. The zero point of the unloaded balance is determined before bodies are weighed. The mass of a body is then obtained by manipulating the standard masses, &c., as described later, until the pointer makes equal excursions to the right and left of the zero position. The balance is then said to be *in equilibrium*, and the mass of the body is then practically equal to the sum of the known masses in the other scale-pan.

* In some balances the beam is not lifted off the central knife-edge, the scale-pans are merely let down on to the base-board.

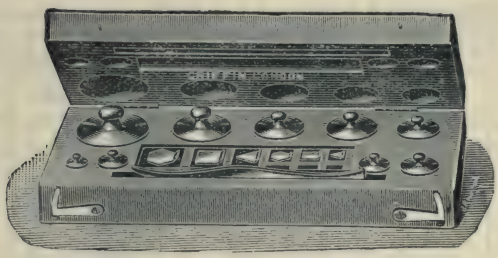


Fig. 19.

26. Set of Metric Weights.—Fig. 19 shows an ordinary box of weights. Its contents are as follows:—

(i.) Brass weights (gilded)—

100, 50, 20, 20, 10, 5, 2, 2, 1 gms.

(ii.) Platinum—

	0·5,	0·2,	0·2,	0·1 gm.
marked	500,	200,	200,	100 mgms.

(iii.) Platinum or aluminium—

	0·05,	0·02,	0·02,	0·01 gm.
marked	50,	20,	20,	10 mgms.

In a "box of weights" each member of (i.) fits into a hole, and there are compartments for the fractional values. Forceps are provided for handling the weights.

By set (i.) above any mass from 1 to 200 grammes may be measured in multiples of the gramme.

By sets (ii.) and (iii.) decimals of a gramme may be measured in multiples of 10 milligrammes.

Examples.—

176·35 = (100 + 50 + 20 + 5 + 1) gms. + (200 + 100 + 50) mgms.

99·08 = (50 + 20 + 10 + 10 + 5 + 2 + 2) gms. + (50 + 20 + 10) mgms.

In *weighing a body* it is best not to add the weights haphazard, but in descending order of magnitude, the equilibrium being tested by releasing the balance after each additon. Consider the following:—

Weights in pan.	Equilibrium test.
100	too little
100 + 50	too much
100 + 20	too much
100 + 10	too little
100 + 10 + 5	too little
100 + 10 + 5 + 2	too much
100 + 10 + 5 + 1	too much

Thus the weight is greater than 115, less than 116. The fraction can be found by adding the smaller weights in a similar manner.

27. Rules to be observed in using a Balance.—

(1) Before commencing to weigh, see that the pans are clean and dry, and the stirrups not dislodged; also that the plummet or spirit level shows the balance to be correctly levelled. Use a camel-hair brush for dusting the scale-pans, &c., if necessary.

(2) *While weighing* observe the short scale of the balance from a position directly in front, and about two feet off. Press the left hand on the balance base, work the arrestment handle with the right. *Hold the handle until it is turned completely over to the right or left.* Release or arrest the balance slowly and without jerking.

(3) *Determine the zero or resting point* of the unloaded balance (§ 25).

(4) Place the box of weights at the right-hand side of the balance; the body to be weighed in the left-hand pan; add the weights to the right-hand pan. *On no account should weights be touched by the hand. The balance must be arrested when a weight is to be added to or taken from the pan.*

(5) A body must not be weighed when hot. All substances liable to injure the pans must be put in appropriate vessels.

(6) Add the known weights in descending order of magnitude (§ 26). The larger weights should be put in the centre of the pan. On no account must the balance be loaded with a weight greater than the maximum it is constructed to carry. (This is sometimes stamped on the instrument.)

(7) If the balance does not swing when released, either arrest and release again, or, by moving the hand in the neighbourhood, beat some *air* down on a pan. *The pointer must not be touched.*

(8) When equilibrium (referred to the zero point of the balance) has been obtained, sum up the weights in the scale-pan, and confirm by observing what spaces in the box are empty. Finally, *replace the weights in the box.*

28. Requirements of a Balance.—It is important that a balance should be

(1) *true*, that is, the weight in one pan should be equal to that in the other;

(2) *stable*, that is, whatever the load, within limits, the balance should vibrate about its original resting point or nearly so;

(3) *sensitive* or *sensible*, that is, a very small overweight should produce a perceptible displacement of the resting point.

A balance should swing somewhat quickly (10 to 15 secs.) in order that weighing may be done rapidly.

29. The Steelyard is a lever balance with unequal arms In Fig. 20 the beam, AB, is movable about a knife-edge, C fixed near one end. From B a scale-pan is suspended, in this the body to be weighed is placed; the movable weight, P, is pushed along the arm, CA, which is graduated and numbered, so that the division at which P rests when there is equilibrium indicates the mass of the body in the scale-pan in lbs., &c.

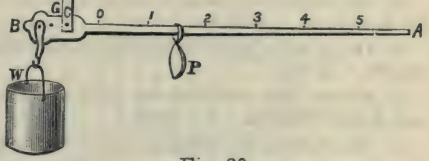


Fig. 20.

Sometimes the scale-pan containing the body to be weighed is adjusted on the graduated arm, and a weight is kept at one position on the short arm.

30. The Spring Balance.—In this the weight of a body is measured by the extent to which it lengthens a spring. A simple form is shown in Fig. 21, the right-hand figure being a front view and the left-hand figure a sectional side view. The tube, *B*, moves easily inside another, *A*. A spring connects the ends of the tubes. A plate, *C*, is fixed to the outer tube. The plate is slotted: an index, *I*, fixed to the inner tube passes through the slot. When the spring is stretched, the index moves downwards, the extension (which is proportional to the load) being indicated on the scale graduated on the plate. The scale (one of equal parts) is *direct reading;* it is numbered so that the value of the weight in lbs., &c., is indicated at once.

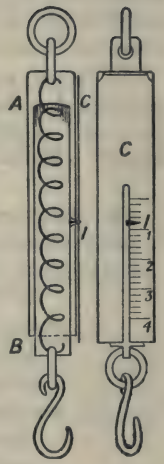


Fig. 21.

By using springs of different lengths and thicknesses, a wide range of weights may be measured. The spring balance is far less sensible than a lever balance. It has, however, the advantages of being direct reading, quick, compact, and portable.

Summary.—Chapter IV.

1. The mass of a body is the "quantity of matter" which it contains. (§ 22.)

2. The weight of a body is the force with which that body is attracted by the Earth. (§ 22.)

3. At the same place the weights of all bodies are proportional to their masses. (§ 22.)

4. Masses (and weights) are most accurately measured by a balance and box of weights. (§§ 23–28.)

5. The balance and weights should be treated with great care. (§ 27.)

6. Weights may be more readily, but not so accurately, determined by means of a steelyard or spring balance. (§§ 29, 30.)

CHAPTER V.

AREA, VOLUME, AND THEIR MEASUREMENT.

Area and its Measurement.

31. Area means "surface-extent." The area of a surface depends upon its length, its breadth, and its shape. In the Tables, the numbers in "square measure" are the squares of the corresponding numbers in "long measure."

32. Areas of Simple Geometrical Surfaces. — These can be calculated if their appropriate dimensions are known.

(1) The area of a **rectangle** is found by multiplying the *number* of inches (or centimetres, &c.) in its length by the number of inches (or centimetres, &c.) in its breadth. The product gives the area in square inches (or square centimetres).

Fig. 22 shows the case of a rectangle *ABDC*, 4 units long and 3 units wide. Its area is obviously 12 square units. Fig. 23 shows the case

Fig. 22.

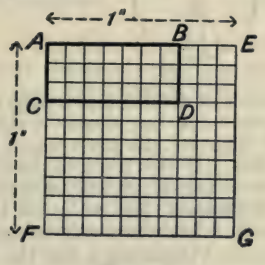


Fig. 23.

of a rectangle *ABCD*, ·7 inch long and ·3 inch wide. Its area is obviously $\frac{21}{100}$ or ·21 square inch.

If A stand for area, l for length, and b for breadth, then

$$A = l \times b.$$

(2) A **square** is a particular form of a rectangle in which the length is equal to the breadth. The area is therefore given by the equation $A = l^2$.

(3) The area of a **parallelogram** (Fig. 24) is found by multiplying the base (l) by the perpendicular height (h), *i.e.* $A = l \times h$.

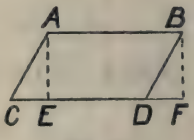


Fig. 24.

For the parallelogram $ABDC$ (Fig. 24) is equal to the area of the rectangle $AEFB$, *i.e.* to $AB \times AE$ or $l \times h$.

(4) The area of a **triangle** (Fig. 25) is found by taking half the product of the base (b) and the perpendicular height (or altitude), *i.e.*

$$A = \tfrac{1}{2}h \,.\, b.$$

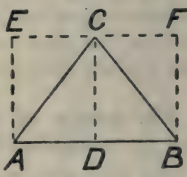


Fig. 25.

As indicated in Fig. 25, the area of the triangle ABC is half the area of the rectangle $ABFE$ on the same base AB and of the same height EA.

(5) The area of a **circle** is found by multiplying half the circumference by the radius.

Fig. 26.

Fig. 27.

A circle can be divided into a large number of triangles (Fig. 26), the base of each being very nearly a straight line; and the greater the number of triangles, the more nearly will their bases approximate to straight lines. If the triangles which make up the whole circle be arranged as in Fig. 27, it will be seen that the height of this parallelogram is practically the radius of the circle, and the base of the parallelogram is half the circumference of the circle; hence

the area of the circle = half the circumference × the radius.

If r = radius, d = diameter, and A = area, we have

$$\text{circumference} = \pi d = 2\pi r; \qquad (\S\ 21)$$

$$\therefore \quad A = \frac{\pi d}{2} \cdot \frac{d}{2} = \frac{\pi d^2}{4}, \quad \text{or} \quad A = \frac{2\pi r \,.\, r}{2} = \pi r^2.$$

The following Table summarizes the above results, and gives a few others without proof.

Area of a **square**	= length × length = l^2.
Area of a **rectangle**	= length × breadth = lb.
Area of a **parallelogram**	= base × height = lh.
Area of a **triangle**	= $\frac{1}{2}$ base × height = $\frac{1}{2}lh$.
Area of a **circle**	= π × square of radius = πr^2.
Area of an **ellipse**†	= π × product of semi-axes = πab.
Area of the curved surface of a **right cylinder** or the side faces of a **right prism***	= perimeter of base × height.
Area of the curved surface of a **right cone***	= $\frac{1}{2}$ perimeter of base × slant height.
Area of a **sphere***	= 4π × square of radius = $4\pi r^2$ = $\frac{2}{3}$ of the whole surface of the right cylinder which just encloses it.

The area of a triangle may also be found by the following rule: — Let a, b, c be the sides, and s half their sum (*i.e.* $2s = a+b+c$); then

$$A = \sqrt{\{s(s-a)(s-b)(s-c)\}}.$$

33. Measurement of Area.—It is possible to measure area directly, but as a rule the result is more easily obtained by calculation from measured lengths.

Method I.—If the figure is regular or simple apply the formula given in the previous section. If the figure is bounded by straight lines, it may be divided into triangles and the aggregate area of these found.

Exp. 10.—Draw a triangle ABC, given $AB = 2$ ins., $BC = 2{\cdot}5$ ins., $CA = 3$ ins. By means of set squares draw AD, BE, CF respectively, perpendicular to BC, CA, AB. Measure the length of each perpendicular and *evaluate* the following:—

(i.) $\frac{1}{2}BC.AD$; (ii.) $\frac{1}{2}CA.BE$;

(iii.) $\frac{1}{2}AB.CF$; (iv.) $\sqrt{s(s-a)(s-b)(s-c)}$.

Each result gives the area of the triangle, 2·48 sq. ins.

* For definitions see § 35.

† Any section of a circular cylinder which is *not* perpendicular to the axis of the cylinder is an ellipse.

Exp. 11.—Construct the quadrilateral $ABCD$ from the following data:—$AB = 1{\cdot}5$ ins., $BC = 1$ in., $CD = 1$ in., $DA = 2$ ins., $AC = 2$ ins. Measure BD.

Find the area of the quadrilateral by drawing perpendiculars from A and C on BD and thus finding the areas of the triangles BAD, BCD. Also draw perpendiculars from B and D on AC and thus find the areas of the triangles ABC, ADC. Test the accuracy of your results by comparing them with the correct answer, viz. 1·696 sq. ins.

Method II.—The area of any figure, whether bounded by straight lines or not, can be found approximately by drawing the figure on squared paper and counting the number of squares it encloses.

If, however, the figure is not entirely made up of whole squares (as the circle in Fig. 28) some attempt must be made to estimate the area of the "broken squares" included within the figure. The following rule works on a principle of averages, and usually gives approximately accurate results*:—

Rule.—*Portions of squares greater than one-half should be counted as whole squares; portions less than one-half should be counted as nothing; portions exactly one-half should be counted as such.*

Exp. 12.—*Determine the area of a circle of radius* 1 *inch.*

Using tenth-inch squared paper, draw a circle with centre O and radius 1 inch (Fig. 28).

In this case it may be assumed that the lines XOX', YOY' divide the circle into four equal areas; accordingly it is only necessary to count the squares in one of these areas, say the quadrant XOY.

The "broken squares" occur round the circumference and are to be counted 1, 0, or $\frac{1}{2}$, according as they are greater than, less than, or equal to half a square.

* This rule is of no value if the number of squares included within the figure is small—say, less than twenty-five. If the number of squares is large—say, greater than a hundred—the error is seldom greater than two squares; but the rule is more reliable for *regular curves* than for any other type of figure. It is obviously very easy to draw a rectangle for which the rule completely fails.

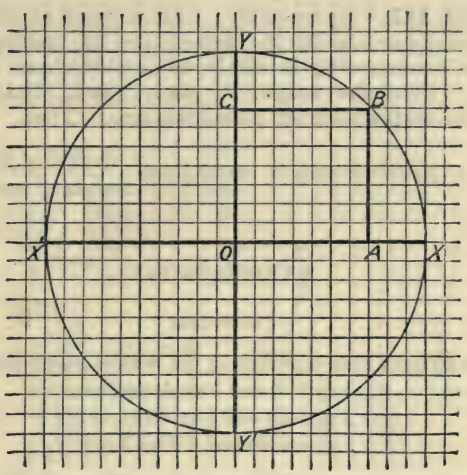


Fig. 28.

To count the squares :—

Number of squares in $OABC$ is (7×7) or	49
Number of squares in the three rows above BC are respectively	$6\frac{1}{2}$, 5, 3
Number of squares in the three columns to the right of AB are respectively	$6\frac{1}{2}$, 5, 3
Total number of squares in quadrant XOY =	78

Thus area of circle $= 4 \times 78$ squares.

$= 4 \times 78 \times \frac{1}{100}$ sq. inches.

$= 3{\cdot}12$ sq. inches.

NOTE.—The *correct* answer (obtained by calculation, using the formula $A = \pi r^2$) is 3·14 sq. inches.

If the figure is already drawn on other paper take a tracing of it and lay the tracing paper on top of the squared paper.

Method III.—*By weighing.* Cut a piece of tin plate or foil or cardboard to fit the figure or its tracing: also a rectangle, whose area is easily calculated, from the same material. Find the weight of each piece. Calculate the area of the rectangle from its dimensions. Then

$$\text{area of figure} = \text{area of rectangle} \times \frac{\text{weight of figure.}}{\text{weight of rectangle.}}$$

EXERCISES IV.

Practical.

1. Draw straight lines representing the relative magnitudes of
   (i.) 1 sq. foot and 1 sq. decimetre;
   (ii.) 1 sq. inch and 1 sq. centimetre;
   (iii.) 2 sq. inches and a 2-inch square.

2. Find the area of a parallelogram whose sides are 8 centimetres and 10 centimetres, its acute angle being $\frac{2}{3}$ right angle. (Draw the parallelogram and find the area by measuring the lengths of the perpendicular between two opposite sides.)

3. Draw a triangle whose sides are 3, 4, and 5 centimetres. Do you observe any striking feature? Find its area.

4. Draw a triangle whose sides are 4, 5, and 7 centimetres on squared paper. Find its area
   (*a*) by measuring its height;
   (*b*) by counting squares;
   (*c*) by cutting it out and weighing, comparing its weight with the weight of a square of 3 centimetres side, cut out of the same paper.

5. Draw a square of 6 centimetres side on a piece of millimetre-squared paper; in it inscribe a circle of 3 centimetres radius. Count the number of square millimetres in the circle and so compare the areas of the circle and square. Paste the paper upon thin cardboard, carefully cut out the square and weigh it on a balance. Now cut the circle out and weigh it. Find in this way the ratio of the area of the circle to that of the square.

6. Find the area of the curved surface of a right cylinder by folding a paper rectangle around it. Then find the total area of the curved surface of the cylinder.

7. Find the area of the face of a penny correct to 1 per cent. The impress of the penny can be taken on thin lead foil. This can be cut out and weighed, and the weight compared with the weight of a known rectangular area of the same foil. Check by measuring the diameter and calculating from the formula $\frac{\pi d^2}{4}$.

8. A party of men surveyed an irregular field, and at the close of their outdoor labour their note-book had, as an account of their work, the following table and Fig. 29 :—

Links.

	To *U* 543	
	T 491	138 to *V*
113 to *Z*	*S* 435	
	R 204	78 to *W*
185 to *Y*	*Q* 168	
	P 137	286 to *X*
	From *O*	

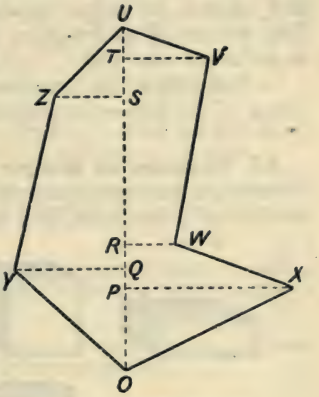


Fig. 29.

Plot the field from these measurements (squared paper may be used), and find the area in agricultural units (*a*) graphically, (*b*) solely by calculation. (NOTE. — 100 links = 1 chain = 22 yards.)

CALCULATIONS.

9. Find the area of the triangle in Question 4, using the formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}.$$

10. Find the area of the field in Question 8 by dividing it into triangles and summing their individual areas.

11. Four pounds of sand are sprinkled evenly over a circular brass plate 6 inches in diameter. How many grammes are there on each square centimetre?

12. What is the area of a circular ring when the radii of the two circles are 7 inches and 1 foot 2 inches respectively? (Take $\pi = \frac{22}{7}$.)

13. If a foot-rule, by which the sides of a rectangle are measured, is too long by ·006 inch, find by what decimal of the calculated area the true area will exceed the calculated.

14. A certain district has been mapped on the scale of 6 inches to a mile. What is the actual area in acres of a field which is represented on the map by $\frac{1}{2}$ sq. inch?

Volume and its Measurement.

34. The **Volume** of a body is defined in § 2 as the measure of the space it fills.

If the body is a regular or simple solid, its volume can be calculated if its shape and dimensions (*i.e.* its length, breadth, and height, or other approximate dimensions) are known. The volume of an irregular body can only be determined by experiment (see § 38).

In the tables the numbers in "cubic measure" are the cubes of the corresponding numbers in "long measure."

35. Volumes of Simple Geometrical Solids. — Fig. 30 illustrates the rectangular block, cube, tetrahedron, pyramid, wedge, cylinder, cone, and sphere.

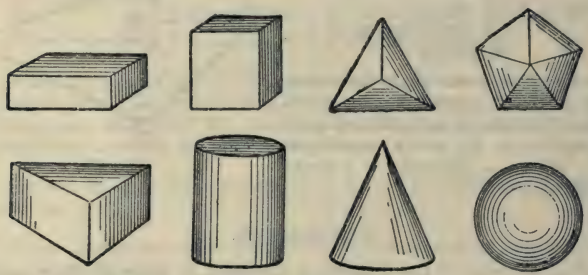


Fig. 30.

Note that the tetrahedron and pyramid are represented as seen from above.

A **parallelepiped** is a solid figure bounded by three pairs of parallel plane faces. The rectangular block is a particular case in which all the faces are rectangles. A cube is a particular form of the rectangular block in which all the faces are equal.

A **prism** is a solid figure enclosed by plane figures, two of which (the ends) are parallel and equal in all respects to each other; the others, called the side-faces, are parallelograms. The ends may be any shape as long as they are bounded by straight lines. A parallelepiped is a particular case in which the ends are parallelograms. The **wedge** is a triangular prism.

A prism is said to be **right** if its end-faces are perpendicular to its side-faces, or **oblique** if they are not so.

A **pyramid** (Fig. 31) is a pointed solid figure enclosed by a **base** of any rectilinear shape and a number of triangles which have a common vertex called the **vertex** of the pyramid, and whose bases are the sides of the base of the pyramid.

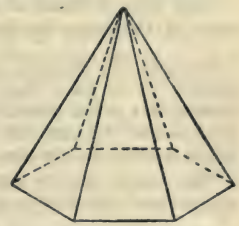


Fig. 31.

A **tetrahedron** is a triangular pyramid.

A **cylinder** is a special case of a prism in which the ends are circles. (A circle may be regarded as a figure contained by an immense number of exceedingly small straight lines). The straight line joining the centres of the two ends is the **axis.**

In the same way, a **cone** is a special case of a pyramid in which the basis is a circle. The axis of a cone is the straight line joining the vertex to the centre of the base.

A **right** cylinder is one in which the axis is perpendicular to each of the ends. A **right** cone is one in which the axis is perpendicular to the base.

A **sphere** is the solid figure generated by a semi-circle rotating about its diameter. All plane sections of a sphere are circles.

(1) The volume of a **rectangular block** (*i.e.* rectangular parallelepiped) is found by taking the product of the length, the breadth, and the height.

Fig. 32 shows a rectangular block of length 5 units, breadth 3 units, and height 2 units, divided into unit cubes. It contains 2 layers of cubes; each layer contains 3 rows of cubes, and each row consists of 5 cubes. Hence the volume is obviously

$5 \times 3 \times 2$, *i.e.* 30 cubic units.

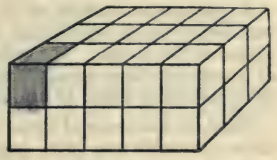


Fig. 32.

If V stand for volume, l for length, b for breadth, h for height, then $V = l \times b \times h$.

The length, breadth, and height must all be expressed in terms of the same unit. If the unit is an inch, the volume is in cubic inches; if a centimetre, the volume is in cubic centimetres.

(2) The volume of a **cube** is given by $V = l^3$.

(3) The volume of **any prism** is found by multiplying the area of the base by the perpendicular height.

Suppose the triangle ACB (Fig. 25) is the base of a right triangular prism. It is plain that the volume of this prism is half that of the rectangular prism standing on the rectangle $AEFB$. But the area of ACB is half the area $AEFB$. Therefore the volume of the triangular prism is equal to the area ACB multiplied by the height.

This result can be extended (by a more difficult argument) to all prisms, and therefore to cylinders.

Hence the volume of a **right circular cylinder** is given by the formula $V = \pi r^2 h$, where r is the radius of the circular base.

(4) The volume of **any pyramid** is found by taking one-third of the product of the area of the base by the perpendicular height.

The proof is beyond the scope of this book.

Hence the volume of a **right circular cone** is given by the formula $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the circular base.

(5) The volume of a **sphere** is found by multiplying the cube of the radius by $\frac{4}{3}\pi$.

Suppose the surface of a sphere to be divided into a very large number of small rectilinear figures, and the corners of these figures joined to the centre. The sphere is thus divided into a very large number of pyramids whose small bases are practically flat, and whose altitudes are practically equal to the radius of the sphere.

Thus, if the areas of these bases are a_1, a_2, a_3, &c., the volumes of these pyramids are $\frac{1}{3}a_1 r$, $\frac{1}{3}a_2 r$, $\frac{1}{3}a_3 r$, &c.

Thus volume of sphere $= \frac{1}{3}a_1 r + \frac{1}{3}a_2 r + \frac{1}{3}a_3 r + \ldots$

$= \frac{1}{3}r\,(a_1 + a_2 + a_3 + \ldots)$

$= \frac{1}{3}r \times$ total area of sphere

$= \frac{1}{3}r \times 4\pi r^2$

$= \frac{4}{3}\pi r^3$.

This happens to be just $\frac{2}{3}$ of the volume of the right cylinder which just encloses the sphere. (See § 32.)

Exp. 13.—*Find the volume of a rectangular block of wood or glass.*—Measure its dimensions with a millimetre scale, taking the mean of several values. Suppose the dimensions are 43·4, 27·6, and 15·2 mms. Then the volume = 43·4 × 27·6 × 15·2 cub.mms. Multiplying out, we obtain 18207·168 cub.mms. It would, however, be wrong to give this up as an answer, for the measurements made with the ruler are at best only correct to the nearest tenth of a millimetre; and hence the length measured as 15·2 mms. is probably anything between 15·3 and 15·1 mms. Thus the error in this measurement may be ·1 in 15·2, or roughly $\frac{2}{3}$ per cent. Errors will also occur in the other measurements, but probably to a less extent; for, if we apply the same reasoning to 43·4 as that given above, we see that the error would probably be less than $\frac{1}{2}$ per cent. In any case the answer cannot be credited with an accuracy greater than that obtained in the measurement of the smallest dimension, and, as a rule, the accuracy would be much less owing to the other errors. We therefore write our answer as 18200 cub.mms., and place little reliance on the 2. Of course, errors in one dimension may neutralize errors in another; but this cannot be depended upon.

A *general rule* is to give the same number of significant figures (3 in the above) in the answer as occur in the number representing the least dimension.

In consequence of this rule approximate methods of calculation can be used in nearly all physical work, with a great saving of time and trouble and without sacrificing the accuracy of the result.

Example.—The thickness of a cubical crystal of fluorspar is measured by the screw-gauge. The result is ·061 cm. Find the volume in cubic centimetres.

$$(\cdot 061)^3 = \cdot 000286981.$$

Now in ·061 there are two significant figures, for the cipher does not count; hence the answer should contain only two significant figures. The answer is therefore ·00028 + cub. cm.; and, since the figure after the 8 is a 6 (and nearly a 7), the answer should be given as ·00029 cub. cm.

NOTE.—Ciphers do not count as significant figures unless they *lie between* numbers which are not ciphers. Thus in either of the quantities 3002500 and ·030025 there are five significant figures, viz. 30025.

EXERCISES V.

Practical and Calculations.

1. Find the number of cubic centimetres in a cubic inch. Draw two straight lines on paper to represent their relative magnitude.

2. Given that there are 2·540 cms. in an inch, express the volume of a cubic decimetre in cubic inches.

3. Find the internal and external volumes of a rectangular box, and from these find the volume of the box correct to within 1 per cent.

4. Obtain the volume of a cylindrical canister. Measure the length and diameter in centimetres. Calculate the volume in cubic centimetres.

5. Obtain the volume of a spherical ball. Use glass or stone marbles, and measure the diameter as in Exp. 5.

6. Find the volume of a wooden wedge, a glass prism, a cone (a conical wine-glass will do) in cubic centimetres and in cubic inches. Hence determine the ratio $\frac{\text{cubic inch}}{\text{cubic centimetre}}$, and compare it with that found by calculation.

Capacity.

36. The **capacity** of a body is its internal volume. It could be measured in the ordinary units of volume, viz. the cubic foot or the cubic centimetre; but it is more usual to have a special name for it.

Thus in England the unit of capacity is the gallon. This is defined as the volume of 10 lbs. of water at 62° F. This is sometimes remembered by the rhyme

> "A pint of clear water
> Weighs a pound and a quarter."

In ordinary units a gallon = 277·46 cubic inches. In the metric system the unit of capacity is the **litre** (§ 10). The "litre" is merely a name used instead of "cubic decimetre" when referring to the volume of a quantity of fluid or to the holding power of a vessel. The basis of all calculation in this part of the subject is that 1 kilogram of water at 4° C. occupies 1 litre (see § 10).

37. Measuring Vessels (Fig. 33).—The volume of a quantity of liquid is determined by some form of measuring vessel. For scientific purposes these are usually made of glass; trade measures are of wood (bushel, &c.) or metal (quart, &c.). *Measuring flask, F:* when filled up to the mark *m*, it contains a definite volume (1000, 500, or 250 cubic centimetres, &c.) of a liquid. *Measuring or graduated cylinder, C:* the marks on the side show the volume between a division and the bottom; these are graduated in *cubic centimetres*, one-eighth of an ounce, one-tenth of a cubic inch, &c.

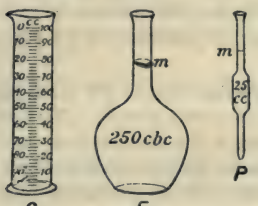


Fig. 33.

Pipette, P: when filled to the mark *m*, it holds a definite volume (100, 75, 50, or 25 cubic centimetres) of fluid. A pipette is useful for adding or removing a small quantity of liquid to or from a vessel. The pipette is filled by suction.

Burette, B (Fig. 34): a narrow tube with a tap or pinch-cock (indiarubber tube and clip), by means of which the liquid may be let out; these are frequently graduated in tenths of a cubic centimetre, and read downwards, so that the volume of liquid delivered can be measured. Burettes are clamped in a vertical position to a wood or iron stand.

Fig. 34.

Readings of measuring vessels should not be hurriedly done; the liquid requires a little time to drain down from the walls. The reading, especially of the burette, is liable to parallax error. To avoid this, place the eye level with the surface. The surface is slightly curved; it is usual to read from the bottom of the curve.

Exp. 14.—*To find the volume of a vessel.*—(1) Fill the measuring cylinder up to a mark with water. Note the position. Pour water from it, without spilling, into the vessel whose volume is to be found. Observe how much water is left in the measure. Deduce the volume of water poured out: this equals the volume of the vessel. (1*a*) Fill the vessel with water. Pour the water from it into a measuring cylinder, and note the volume.

(2) Weigh the vessel (i.) empty, (ii.) full of water. The increase in weight (i.) in grammes is numerically equal to the volume in cubic centimetres, (ii.) in ounces is numerically equal to the volume in thousandths of a cubic foot. The volume in cubic inches is obtained by multiplying the increase of weight in ounces by 1·73.

Exp. 15.—*Make and graduate a measuring vessel.*—(1) Close the small end of a cylindrical lamp chimney with a cork. (To avoid leakage put some pieces of candle wax in the tube, and expose to a fire so that the wax melts and soaks into the cork.) Gum a strip of stamp paper along the outside. Mark every fifth notch of the stamp paper, beginning to count near the cork. Number every tenth notch, 1, 2, 3, &c. Read intermediate notches as decimals. (2) Mark the stem (bind a piece of cotton or fine wire round) of a rough pipette. Fill up to the mark with water (§ 37). Discharge the water into the measuring vessel, allow the pipette to drain, blow out the last drop. Note the reading of the water surface in the vessel. (3) Again fill the pipette, add the water as before to the measuring vessel, and note the height to which it rises. Repeat until the vessel is full. The measuring vessel has thus been *calibrated* or divided into parts of *equal volumes*.

RECORD as below:—

Volumes	1	2	3	4	5	&c.
Reading of water surface ...	.	.	.	.	.	.

38. To find the Volume of a Body (which may be in fragments.—

(A) **By Displacement.** — (1) Put some water into a measuring cylinder. Note the reading of the surface, (i.) before, (ii.) after, the body is introduced. The difference is the volume of the body. The water introduced at the beginning must be sufficient to cover the body.

(2) Place the body in a dry measuring cylinder. Run in water from a burette until the body is completely immersed and the surface stands at a definite division of the measure. Read the burette before and after discharging the water, and calculate the volume delivered. The difference between the volume delivered from the burette and the reading of the measuring vessel is the volume of the body.

(B) A commoner and more accurate method is that which involves the use of the Principles of Archimedes. (See § 48, *et seq.*)

Summary.—Chapter V.

Area.

1. The areas of rectangular figures or of simple geometrical curved figures can be calculated if their appropriate dimensions are known. (§§ 32, 33.)

2. Other areas may be found by (*a*) using squared paper, (*b*) weighing. (§ 33.)

Volume.

3. The volumes of regular and other simple solids can be calculated if their appropriate dimensions are known. (§ 35.)

4. The capacity of a body or vessel is its internal volume: the units are the gallon and the litre. (§ 36.)

5. Common measuring vessels are the measuring flask, the graduated cylinder, the burette, the pipette. (§ 37.)

6. The volume of a vessel can be measured by pouring in water from a measuring vessel. (§ 37.)

7. The volume of an irregular solid can be found by measuring the volume of the amount of water it displaces. (§ 38.)

EXERCISES VI.

Practical.

1. Make, accurately, a cube of paper measuring 2 centimetres each way internally, and dip it into hot paraffin to make it waterproof. Use this cube to graduate a tube into cubic centimetres, marking the divisions on a strip of gummed paper.

2. Counterpoise the 2-centimetre cube previously made, fill it with water, and cut off a piece of lead foil equal in mass to this water. What should the mass of this lead be? Why is it not exactly what you had supposed?

3. Using the graduated tube made in Question 1, find the volume of the given bottle, and calculate how many times a litre would fill it?

4. Obtain the volume of the cylindrical canister—

   (i.) by filling it with water from a measuring vessel, or by filling it with water and emptying the water into the measuring vessel;

   (ii.) weigh the can (*a*) empty, (*b*) full of water.

Compare results with that obtained by direct measurement of its dimensions.

5. Obtain the volume of a spherical ball by displacement in the measuring vessel.

(i.) Use glass or stone "marbles." Find the displacement for a number of them, and calculate the displacement of a single one.

(ii.) Find diameter by direct measurement and use $V = \frac{4}{3}\pi r^3$.

6. Find the volume of (*a*) a pair of scissors or something else more or less complicated in shape, (*b*) a piece of india-rubber tubing; (*c*) a lump of sugar.

7. Obtain the volumes of a medicine bottle marked in tea or table spoons. Fill from the measuring vessel up to one of the marks, then up to another, &c. Calculate the mean value of the tea or table spoon.

8. You are given a sphere and a block of wood containing a cylindrical hole into which the sphere just fits so that its surface is flush with the face of the block. Find the relation between the volumes of the sphere and the cylinder. The volume of the cylinder being $\pi r^2 \times 2r$, *i.e.* $2\pi r^3$, what formula expresses the volume of a sphere?

9. Find the radius of a ping-pong ball. Calculate its volume and confirm it by the displacement method.

10. I have lost all the masses under 2 decigrams from my box of weights. How can I make a make a mass of 10 milligrammes from a piece of thin aluminium wire?

11. Given a number of lead shot of equal size, find the mass of a single shot to 1 milligramme.

Calculations.

12. A cask holds $4\frac{1}{2}$ gallons. How many litres will it hold?

13. How many litres of water can be poured into a cylindrical can 50 centimetres high and 40 centimetres in diameter?

14. What is the mass in kilogrammes of the water in a cube whose side is 1 metre?

15. A rectangular box measures 25 cm. × 16 cm. × 12 cm. How many pounds of water can it hold?

16. A cylindrical can is 15 inches high and 12 inches in diameter. How many kilogrammes of water can be poured into it?

CHAPTER VI.

DENSITY AND SPECIFIC GRAVITY.

This chapter deals with the density and specific gravity of solids and liquids, and some experimental methods for their determination.

39. Density of Solids and Liquids.—

Exp. 16. — Weigh equal-sized solid cubes or balls made of wax, wood, metal, or other materials. The weights are different.

Exp. 17.—Take a small bottle. Weigh it (i.) empty, (ii.) *full* of water, (iii.) *full* of methylated spirits. Subtracting the first result from each of the others we obtain the weights of the water and the methylated spirits. The volumes of the two liquids are equal (the bottle being full in each case), but the weights will be found unequal.

Deduction.—Since the weights of bodies are proportional to their masses, it follows that *equal volumes of different substances may have different masses.*

DEFINITION. — **The mass of unit volume of any substance is called the density of that substance.**

The number which measures the density of a substance depends not only on the substance, but also on the choice of units of length and mass. Thus—

the density of water = **1** gramme per cubic centimetre
= **62½** pounds per cubic foot
= **1000** ounces per cubic foot.

Example.—Find the mass of sea water in a rectangular tank whose base measures 3 ft. by 2 ft., filled to a height of 18 ins., if the density of sea water is 64 lbs. per cubic foot.

The mass of 1 cub. ft. of sea water is 64 lbs.

The volume of water in the tank = $(3 \times 2 \times 1\frac{1}{2})$ cub. ft. = 9 cub. ft.

Hence the mass of the water in the tank = (64×9) or 576 lbs.

40. Density of Gases. — Gases, being material substances, have weight, although their density is very small compared with that of most solids and liquids.

Thus, 1 cub. in. of water when boiled at ordinary pressure yields about 1 cub. ft. or, more exactly, 1,650 cub. ins. of steam. But matter is indestructible; hence the mass of the steam is equal to that of the water, and its density therefore is only $\frac{1}{1650}$ of the density of water.

41. Measurement of the Density of a Substance.— Take a body composed of the material, measure its mass (by weighing in air) and its volume. Then the

(*density of substance*) = (*mass of body*) ÷ (*volume of body*).

Exp. 18.—*Obtain the volume and mass of a block of wood or glass, a piece of coal, a stone, &c. Calculate the density of each.*

The masses can be determined in grammes by the balance, the volumes in cubic centimetres by calculation from their dimensions or by the measuring vessel. Divide the mass of each by its volume: the quotient is the density of the material in grammes per cubic centimetre.

42. Specific Gravity. — Specific gravity measures the same property as density, but in a different manner, viz.: by comparing the body with a standard substance. The standard substance (except in the case of gases) is water at a temperature of 4° C. Thus, if a substance is 6 times as heavy as water (comparing equal volumes), we say that its specific gravity is 6.

DEFINITION.—**The specific gravity (sp. gr.) of a substance is the ratio of the mass of any volume of the substance to the mass of an equal volume of water at 4° Centigrade.***

Water at 4° Centigrade is chosen as the standard body, as it is at its greatest density at that temperature.

Caution.—Specific gravity is therefore a *number*, not a *weight* nor a *mass*.

As the masses of two bodies are in proportion to their weights, we may also define

The specific gravity of a substance as the ratio of the weight of any volume of the substance to the weight of an equal volume of water at 4° Centigrade.

* Specific gravity is sometimes wrongly spoken of as *specific density* or *relative density*.

Specific Gravity of Gases.—Most gases are very light compared with water; their specific gravities when so compared would be very small. It is therefore often convenient in dealing with them to take either air (the commonest gas) or hydrogen (the lightest gas) with which to compare them.

43. Relation between the Weight, Volume, and Specific Gravity of a Body.—We can now find the weight of any volume of a substance of given specific gravity.

Examples.—(1) What is the weight of a cube of copper whose side is 2 ft., the specific gravity of copper being 8·9?

The volume of the copper is $(2\times2\times2)$ or 8 cub. ft.

Now 8 cub. ft. of water weigh 8000 oz., and 8 cub. ft. of copper weigh 8·9 times as much, *i.e.* weigh $(8000\times8{\cdot}9)$ oz., which

$$=(500\times8{\cdot}9)\text{ lbs.}=4450\text{ lbs.}$$

(2) Find, in ounces, the weight of a cubic inch of lead, taking the specific gravity of lead to be 11·4.

Weight of a cubic foot of water = 1000 oz.

$\therefore$ weight of a cubic inch of water, *i.e.* of $\frac{1}{1728}$ cub. ft. $=\frac{1000}{1728}$ oz.

But a cubic inch of lead weighs 11·4 times as much;

$\therefore$ weight of a cubic inch of lead $=\dfrac{1000\times11{\cdot}4}{1728}=6{\cdot}59$ oz. approx.

(3) Find the density and the specific gravity of a substance if a cubic yard weighs 1080 lbs.

27 cub. ft. weigh 1080 lbs. Thus 1 cub. ft. weighs 40 lbs.

Hence the density is 40 lbs. per cubic foot.

1 cub. ft. of water weighs 1000 oz. Hence the specific gravity of the substance is

$$\frac{40\text{ lbs.}}{1000\text{ oz.}}=\frac{40\times16}{1000}=\cdot64.$$

EXERCISES VII.

Calculations.

1. What is the density of a body whose mass is 4 lbs. and volume 20 cub. ins.?

2. What is the volume of a body whose mass is 8 lbs. and density 16 lbs. per cub. ft.?

3. Find the specific gravity of copper if 5 cub. ft. weigh $24\frac{3}{4}$ cwts.

4. What is the weight of 10 cub. ft. of a body whose specific gravity is 6·4?

5. The specific gravity of gold being 19·2, and that of lead being 11·2, find the ratio which the mass of 5 cub. ins. of lead bears to the mass of 7 cub. ins. of gold.

6. What are the specific gravities of substances of which

(i.) 1 cub. in. weighs 1 oz.; (ii.) 1 cub. yd. weighs 1 ton?

7. If 28 cub. ins. of water weigh 1 lb., what will be the specific gravity of a substance 20 cub. ins. of which weigh 3 lbs.?

8. If 75 cub. cms. of a body weigh 90 gms., what is its specific gravity?

9. If 1 cub. in. of the standard substance weigh ·45 of 1 lb., what is the weight of 1 cub. yd. of a substance whose density is 5?

44. Determination of Specific Gravity. — In the practical determination of the specific gravity of a substance, the two chief measurements to be obtained must be carefully kept in mind, viz.:

(i.) the weight of some portion of the substance;

(ii.) the weight of an equal volume of water.

The first of these is readily obtained, and it is to the various methods of estimating the second that particular attention must be given.

In finding the specific gravity of a liquid by means of the specific gravity bottle, both measurements are obtained *directly*.

45. The Specific Gravity Bottle (Fig. 35) is much used for finding the specific gravities of solids and liquids. It is constructed for the purpose of weighing exactly equal volumes of different liquids, and it consists of a glass flask having a tightly fitting stopper through which runs a very fine hole (*ab*).

Fig. 35.

In using the bottle, it is completely filled with the liquid to be weighed, and the stopper is then pushed in. The superfluous liquid overflows through the hole *ab* and is wiped off; so the bottle, when filled in this way, always contains the same volume of liquid.

Better even than the bottle are **U**-tubes (called pyknometers) with capillary ends, because these tubes are more easily filled and cleansed than a bottle.

Exp. 19.—*Make a specific gravity bottle* (Fig. 36).—

Take a piece of $\frac{1}{2}$-inch soft-glass tubing about 10 inches long. Soften an inch of the tube about 3 inches from one end in the flame, and when the walls have become greatly thickened remove the tube from the flame and gently pull the ends of the tube out about 2 inches, so that the neck becomes fairly

narrow. Now heat one end in the flame, and, by using a bit of odd glass, pull it out until it is sealed. Then heat the closed end for about 2 inches, and when sufficiently hot blow a large bulb, taking care that the walls are firm. Finally, give a flat bottom to the bulb by softening it in the flame, and then cut off the other end of the tube. At the narrowest part of the neck make a mark *f* with a file. At each filling the surface of the liquid must be adjusted to this mark. To facilitate this operation, use a home-made pipette, made by drawing out the end of the piece of tubing left after making the bulb.

Fig. 36.

46. To find the Specific Gravity of a Liquid by the Specific Gravity Bottle.—

Exp. 20.—As an example, suppose we wish to find the specific gravity of **sulphuric acid.**

(i.) Weigh the empty bottle, clean, and dry. (Suppose the weight $W = 10$ gms.)

(ii.) Fill the bottle with the acid, adjust to the mark, and weigh. (Suppose the weight $W_1 = 65$ gms.)

(iii.) Remove acid from bottle, wash out several times with clean water, fill to the mark with distilled water,* and weigh. (Suppose the weight $W_2 = 40$ gms.)

It follows that

(*a*) Weight of the bottleful of sulphuric acid
$= W_1 - W = 65$ gms. $- 10$ gms. $= 55$ gms.

(*b*) Weight of the bottleful of water
$= W_2 - W = 40$ gms. $- 10$ gms. $= 30$ gms.

(*c*) Hence specific gravity of sulphuric acid
= wt. of sulphuric acid ÷ wt. of an equal vol. of water

$$= \frac{W_1 - W}{W_2 - W} = \frac{55}{30} = 1{\cdot}83.$$

Any other liquid could obviously be treated in the same manner.

Specific gravity bottles are usually constructed to hold 10, 20, 25, 50, or 100 gms., or 250, 500, or 1000 grains, of water, and when this is the case there is no need to weigh the bottle filled with water.

* The temperatures of water and acid should be equal or as nearly equal as possible. Rise of temperature is accompanied by decrease of density, and different liquids decrease at different rates.

47. To find the Specific Gravity of a Solid Substance by means of the Specific Gravity Bottle—

(1) When the body is in fair-sized fragments.

Exp. 21.—As an example, suppose we wish to find the specific gravity of some **lead shot.**

(i.) Weigh the shot. (Suppose the weight $M = 100$ gms.)

(ii.) Weigh the bottle full of water. (Suppose the weight $W_1 = 40$ gms.)

(iii.) Place the shot in the bottle. Hold the bottle on the slant and slowly rotate it to dislodge air bubbles. Adjust the water surface to the mark, clean and dry the outside, weigh. (Suppose the weight $W_2 = 131$ gms.)

If we subtract W_2 from $M + W_1$, we shall obtain the *weight of the water spilt*, for there is the same weight of lead and glass in (iii.) as in (i.) and (ii.); hence the difference is due to the spilt water only. Thus

weight of water spilt $= 100 + 40 - 131 = 9$ gms.

Also the volume of the spilt water is equal to the volume of the lead shot.

Thus the specific gravity of the lead $= \dfrac{\text{weight of the lead}}{\text{weight of the water spilt}}$

$$= \frac{M}{M + W_1 - W_2} = \frac{100}{9} = 11{\cdot}1.$$

(2) When the body is in the form of numerous small fragments.

Exp. 22. — Suppose the specific gravity of some dry sand is required.

(i.) Weigh the empty bottle, clean, and dry. (Suppose the result $W = 10$ gms.)

(ii.) Half fill the bottle with sand by means of a little paper cone; weigh. (Suppose the weight $W_1 = 50$ gms.)

(iii.) Cover the sand with water, and shake well to get rid of air bubbles. Finally, fill up with water and weigh. (Suppose the weight $w = 64$ gms.)

(iv.) Remove all the sand by shaking and inverting, wash out well, fill up with water only, and weigh. (Suppose the weight $W_2 = 40$ gms.)

Then weight of sand $= W_1 - W = 50 - 10 = 40$ gms.

The bottle will hold $W_2 - W$, *i.e.* 30 gms. of water. When the sand is present, only $w - W_1$, *i.e.* 14 gms. of water, can be poured in. Therefore the sand replaces $(W_2 - W) - (w - W_1)$, *i.e.* 16 gms. of water. Hence specific gravity of sand

$$= \frac{\text{weight of sand}}{\text{weight of an equal volume of water}} = \frac{40}{16} = 2{\cdot}5.$$

If the body is soluble in water, a liquid may be used in which it is insoluble. A common liquid generally available for this purpose is kerosene (*i.e.* paraffin oil).

EXERCISES VIII.

Practical.

1. Using rectangular or cylindrical lumps of glass, wood, iron, &c., find the density of these substances.

2. Measure the volume of the lead in a piece of lead piping, and from this and its density find its mass. Verify by direct weighing.

3. Using a small flask, find the specific gravity of methylated spirits. Repeat with a specific gravity bottle.

4. Find the specific gravity of a saturated salt solution by the bottle.

5. Find the specific gravity of brass (nails), iron (nails) by the bottle.

6. Find the specific gravity of sand by the bottle.

7. Find the specific gravity of salt by the bottle. (Use kerosene in the bottle.)

Calculations.

8. How many cubic centimetres are there in a body which weighs 24 gms. and whose specific gravity is ·18?

9. If 5 cub. ins. of silver weigh as much as 21 cub. ins. of plate glass, and the specific gravity of silver be 10·5, find that of plate glass.

10. What is the mass of a globe of lead $\frac{1}{2}$ of a metre in diameter? (Sp. gr. of lead = 11·35.)

11. The specific gravity of turpentine being ·85, and 1 gallon of water weighing 10 lbs., how many pints of turpentine will weigh $4\frac{1}{4}$ lbs.?

12. A pound of iron is to be drawn into wire having a diameter of ·05 in. What length will it yield, the specific gravity of iron being 7·6?

13. When a specific gravity bottle is full of water, it is counterpoised by 983 gms., in addition to the counterpoise of the empty bottle, and by 773 grs. when filled with alcohol. What is the specific gravity of the alcohol?

14. The weight of a specific gravity bottle when empty is 42 gms., and when full of water and glycerine respectively its weight is 222 gms. and 292 gms. Find the specific gravity of glycerine.

15. A specific gravity bottle holds 154·5 gms. of a liquid whose specific gravity is 1·03, and 108 gms. of ether. Find the specific gravity of ether.

48. The Principle of Archimedes. — When a body lighter than water is dropped into water it floats. If, however, the body is heavier than water, it sinks. This is a fact which we know from everyday experience. Thus a cork floats on water, while a stone sinks to the bottom. If we push a cork down under water, it will again rise to the surface, though the force of gravity on it acts *downwards*.

Therefore we infer that a fluid is capable of exerting an upward force or thrust tending to lift any immersed body to the surface.

We commonly speak of this action as due to the *buoyancy* of the fluid. The upward force is exerted by the fluid on the surface of the solid. Its amount is given by **the Principle of Archimedes,** viz.:

A solid wholly or partly immersed in fluid experiences an upward thrust which is equal to the weight of the fluid which it displaces.

NOTE.—The "displaced fluid" is the fluid which could occupy the space below the surface of the fluid now occupied by the immersed solid. Thus, when the solid is *totally* immersed, the volume of the displaced fluid is equal to the volume of the solid.

49. Experimental Proof of the Principle of Archimedes.—

Case of a body which would sink in water.—

Exp. 23. — (1) *By means of a Spring Balance* (Fig. 37).—

Obtain a cube of metal and a spring balance indicating grammes.

(i.) Measure the cube, and weigh it by suspending it from the spring balance.

(ii.) Immerse the solid in water, and read the balance again.

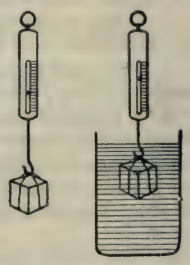


Fig. 37.

Repeat the experiment as often as possible with cubes of different material, and tabulate your results thus:

Weight of solid in air (W) in grammes.	Weight of solid in water (W') in grammes.	$W - W'$.	No. of cub.cms. of solid.

Observation.—Corresponding numbers in columns 3 and 4 will invariably be equal.

Deduction.—Now the number of cubic centimetres in the solid is equal to the number of grammes in an equal volume of water, since 1 cub. cm. of water by definition weighs 1 gm.

Hence the difference in weight when the solid is weighed in air and in water is equal to the weight of the water displaced; and this difference in weight is produced by the upward thrust of the water on the cube.

(2) *By means of a Hydrostatic Balance.*—

The only difference between a hydrostatic balance and an ordinary one is that one of the scale-pans of the former is at a sufficient height to allow a vessel of liquid to be placed under it, and has a hook on its under-side from which any small solid may be suspended by means of a fine thread or wire, and weighed when immersed in the liquid.

Exp. 24.—Pour water into a cylindrical dropping funnel F (Fig. 38 *a*) until it is three-quarters full, let a little water run out (to fill the stem), close the stop-cock, and mark the water-level with a strip of gummed paper P. Clamp the funnel into position underneath the short pan of a hydrostatic balance. On the pan place a small beaker or dish B, and suspend from it by a silk or cotton thread a large glass stopper G. Add weights to the other pan, so that the balance is in equilibrium. Now elevate the funnel until the stopper is partially immersed. The balance is

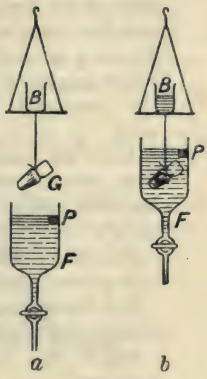


Fig. 38.

no longer in equilibrium, and shows that the water is exerting an upward thrust on the stopper. Elevate the funnel still further (Fig. 38 *b*), until the stopper is completely immersed and is entirely below the mark indicated by the gummed paper P; the water level in the funnel is now above P. Open the cock of the funnel and let water run out from the funnel into the beaker until the surface reaches its former level P.

Now replace B on the pan of the balance. Observe that the *balance is again in accurate equilibrium.* Thus the weight of the water in B exactly neutralizes the upthrust on the stopper. This proves that **the upthrust of the water on the stopper is equal to the weight of water displaced by the stopper.**

Case of a body which would float in water.

Exp. 25.—Take any open vessel B (Fig. 39) filled to the point of overflowing with water: a coffee pot, of which the mouth of the spout is *below* the level of the top will be quite suitable. Take a beaker C and weigh it (let the weight be W): place C so that water emerging from the spout of B may be caught in C. Take any body A which is lighter than an equal volume of water, weigh it, and *gently* lower it by a string into the water until it floats. A quantity of water will overflow whose volume is equal to that of the immersed portion of the solid.

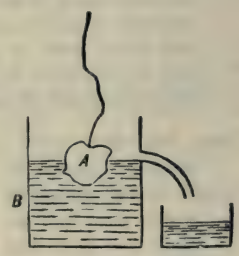


Fig. 39.

After the water has ceased to flow into C, weigh it and its contents again: let the weight be W'.

Then the weight of the water spilled from $B = W' - W$; and this will be found to be equal to the weight of A.

But the upthrust of the water is supporting A, and is therefore equal to the weight of A.

Hence the upthrust is equal to the weight of water displaced.

50. Explanation of Archimedes' Principle.—The explanation of this phenomenon is comparatively easy.

Whether the solid is wholly or partially immersed, the upthrust is produced by the pressures which the liquid exerts on the different parts of the surface of the solid. Now imagine the solid removed, and the space it occupied in the liquid be filled up by some of that liquid. It is obvious that this liquid will remain at rest with no other support than those pressures which were acting on the solid. In other words, the liquid pressures acting on the solid *could support the displaced liquid*, and must therefore produce an upthrust equal to the weight of the displaced liquid.

51. To find the Specific Gravity of a Solid which is heavier than Water.—

Exp. 26.—Suppose we wish to find the specific gravity of the **bronze** in a bronze coin.

(i.) Take a penny, clean it with sand, bore a hole through it near its edge, suspend it by a silk fibre from a spring balance or arm of an ordinary balance, and weigh it. (Suppose the weight $W = 9{\cdot}46$ gms.)

(ii.) Fill a beaker with distilled or clean well-boiled water, and fix it on a stool so that the penny hangs *when the pan is raised*, immersed to the depth of $\frac{1}{2}$-inch in it. Brush off the air bubbles adhering to the penny with a camel-hair brush or a strip of paper. Weigh the penny again. (Suppose the weight $W_1 = 8{\cdot}40$ gms.)

The loss of weight of the penny is $W - W_1$, *i.e.* $9{\cdot}46 - 8{\cdot}40$, or $1{\cdot}06$ gms.

Therefore, by the principle of Archimedes, weight of water displaced by the coin is $1{\cdot}06$ gms. But the volume of the water displaced is equal to the volume of the penny.

$\therefore$ specific gravity of the penny

$$= \frac{\text{weight of the penny}}{\text{weight of an equal volume of water}} = \frac{W}{W - W_1}$$

$$= \frac{9{\cdot}46}{1{\cdot}06} = 8{\cdot}92.$$

52. To find the Specific Gravity of a Solid which is lighter than an equal volume of Water.—

If the solid were suspended by itself, it would float. To make it sink, attach to it a heavy body, called a *sinker*, and weigh the bodies together in air and in water.

Exp. 27.—To find the specific gravity of a piece of **white wax.**

(i.) Take the coin used in the above experiment, warm it, and stick a flat piece of wax to it, taking care that no air bubbles are enclosed. Weigh the two in air. (Suppose the weight = 13·06 gms.)

(ii.) Weigh in water as before, carefully removing all air bubbles. (Suppose the weight = 8·28 gms.)

The weight of the coin in air* is 9·46 gms.;

∴ weight of wax in air is (13·06 − 9·46) or 3·60 gms.

The weight of the coin in water is 8·40 gms. (see Exp. 26);

∴ weight of wax in water is (8·28 − 8·40) or −·12 gms.

Therefore the loss of weight of the wax in water, which is equal to the weight of an equal volume of water, is 3·72 gms.

Therefore specific gravity of wax $= \frac{3{\cdot}60}{3{\cdot}72} = {\cdot}97.$

53. To find the Specific Gravity of a Liquid by finding the apparent weight in it of a body which is denser than the Liquid.

Exp. 28.—To find the specific gravity of glycerine, using a sinker of copper.

(i.) Weigh the copper in air. (Suppose the weight $W = 40$ gms.)

(ii.) Weigh the copper in glycerine. (Suppose the weight $W_1 = 33{\cdot}75$ gms.)

(iii.) Weigh the copper in water. (Suppose the result $W_2 = 35$ gms.)

Then the *weight of glycerine displaced by the copper*

$= W - W_1 = 40 - 33{\cdot}75 = 6{\cdot}25$ gms.

Also, the weight of water displaced by the copper

$= W - W_2 = 40 - 35 = 5$ gms.

But the *volumes of liquids displaced* in each case *are equal*, for the copper displaces its own volume of each.

Hence the specific gravity of the glycerine

$$= \frac{W - W_1}{W - W_2} = \frac{6{\cdot}25}{5} = 1{\cdot}25.$$

* The weight of the coin in air is not *necessary* in this experiment. The wax could be weighed by itself in air.

54. Weight of a Body in Air and *in vacuo*.

In finding the specific gravities of solids, we supposed their weights to be found by weighing them in air with a common balance. If great accuracy is required, it will be necessary either to weigh the bodies *in vacuo*, or to allow for the fact that the bodies, as well as the set of weights employed, all displace more or less air, and therefore the effective* weight of a body in air is less than its true weight by the weight of this displaced air. But the density of air is very small compared with that of most solids and liquids, being $\frac{1}{770}$ of that of water. Hence the weight of the displaced air is in most cases so small a fraction of the weight of the body that no serious error is introduced by neglecting it altogether.

It is easy, however, to make allowance for the displaced air, if necessary. For when a body is placed in one pan of a pair of scales and balanced by weights in the other, the effective weights or resultant forces tending to draw the body and weights towards the ground are equal.

Hence

true weight of body − weight of air displaced by body
= weight of weights − weight of air displaced by weights.

55. The Common Hydrometer (Fig. 40) is adapted for finding the specific gravities of liquids only. It consists of a glass tube or stem *AEG* blown out into two bulbs *B*, *G* at its lower end, and closed at its upper end. The stem and the upper bulb *B* are filled with air, the lower bulb *G* being loaded with mercury or small shot, so that when the hydrometer is in liquid it floats upright with the whole of the bulb and part of the stem submerged.

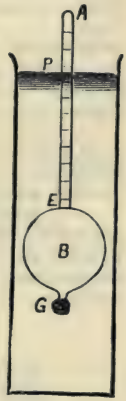


Fig. 40.

Now, floating bodies displace their own weight of liquid. Hence, if this instrument is floated in liquids of different specific gravities, it will have to displace a greater volume when floating in a light liquid than when floating in a heavy liquid; that is to say it will float at a deeper level in the lighter liquid. The stem is provided with a graduated scale. The height to which the liquid rises on the stem is indicated by the scale, and serves to determine the specific gravity of the liquid.

* The effective weight of a body in a fluid is sometimes called its *apparent* weight.

Other methods for the determination of the specific gravity of a liquid are given in § 64.

The Lactometer.—This is a hydrometer designed for measuring the specific gravity of milk. The average specific gravity of milk is about 1·03, so that it is only necessary for the scale to indicate specific gravities from 1·00 to 1·10. The removal of cream from the milk increases the specific gravity and the addition of water lowers the specific gravity, so that it is possible to remove cream and add water and still obtain the right specific gravity. The lactometer is therefore not of much service to the consumer if the reading is nearly correct, but a very high reading would indicate a removal of cream and a very low reading (nearly down to 1·00) would indicate an addition of water. The lactometer is of real use to the dairyman in indicating the relative qualities of milk obtained from different cows.

Table of Specific Gravities.

Solids.

Cork	·25	Marble	2·6
Ice	0·92	Aluminium	2·7
Paraffin wax	0·95	Tin	7·0–7·3
Caoutchouc	·95	Iron	7·0–7·9
Sand	1·4–1·6	Zinc	7·1
Sugar	1·6	Brass	8·5
Magnesium	1·7	Copper	8·9
Gas carbon	1·9	Silver	10·5
Sulphur	2·0	Lead	11·0–11·4
Glass	2·5–2·9	Platinum	21·5

Liquids.

Ether	·73	Olive oil	·92
Alcohol, pure	·79	Sea water	1·026
,, proof spirit	·91	Milk	1·03
Ammonia, strong	·88	Glycerine	1·26
Paraffin oil	·85	Carbon disulphide	1·3
Turpentine	·87	Sulphuric acid, strong	1·85
Benzene	·89	Mercury	13·6

Gases.

	Sp. Gr.	Mass of 1 litre at N.T.P.
Hydrogen	·00009	·09 grm.
Air	·00129	1·293 ,,

Approximate Values.

	Mass of a cubic foot in ounces.	Mass of cub. centimetres in grammes.
Water	1,000	1·0
Atmospheric air	1·3	·0013
Mercury	13,600	13·6

1 gallon of distilled water has a mass of 10 pounds.

Summary.—Chapter VI.

1. The *density* of a body $= \frac{\text{mass}}{\text{volume}}$: $\therefore M = VD$. (§ 39.)

2. The *specific gravity* of a body $= \frac{\text{weight of body}}{\text{wt. of equal vol. of water}}$. (§ 42.)

3. The relation between the volume, weight, and specific gravity of a body is $W = VSw$,
where w is the weight of unit volume of the standard substance. (§ 43.)

4. With the *specific gravity bottle*, the specific gravity of a *liquid*
$= \frac{\text{weight of liquid which fills the bottle}}{\text{weight of water which fills the bottle}}$. (§ 45.)

5. The *specific gravity bottle* can also be used to find the specific gravity of a solid. (§ 47.)

6. *Principle of Archimedes.*—A solid immersed in fluid loses as much of its weight as is equal to the weight of the fluid it displaces. (§ 48.)

7. If a body floats,
its weight = weight of fluid displaced. (§ 48.)

8. If a body does not float, and is supported by a thread,
apparent weight = real weight − weight of fluid displaced. (§ 48.)

9. The specific gravity of a solid which does not float in water is given by the formula $\frac{\text{weight of solid}}{\text{loss of weight of solid in water}}$. (§ 51.)

10. To find the specific gravity of a solid which floats in water, use a sinker;
$\text{specific gravity} = \frac{\text{weight of solid}}{\text{loss of wt. of (solid + sinker)} - \text{loss of wt. of sinker}}$.
(§ 52.)

11. To find the specific gravity of a liquid, use a sinker;
$\text{specific gravity} = \frac{\text{loss of weight of sinker in given liquid}}{\text{loss of weight of sinker in water}}$. (§ 53.)

12. If a hydrometer is floated in two different liquids in turn, it floats *deeper* in the *lighter* liquid. (§ 55.)

EXERCISES IX.

Practical.

1. Find the specific gravity of brine or methylated spirit.
(i.) by the specific gravity bottle;
(ii.) by weighing a body of known density in it, *e.g.* a penny.

2. Find the specific gravity of wax by weighing it in air and in methylated spirit.

3. Determine the specific gravity of glass, sulphur, iron, copper, lead, aluminium, alcohol, ether, &c. (Check the results by reference to the table on p. 60.)

4. Find the specific gravity of a cork. Use a lump of copper as a sinker.

5. Find the mean thickness of a coin or a piece of metallic foil by measuring its area in square centimetres, and its weight in air and water.

6. Find diameters of lengths of wire of various metals by measuring their lengths and their weights in air and in water.

7. Find by the common hydrometer the specific gravity of (i.) concentrated sulphuric acid, (ii.) saturated copper sulphate, (iii.) saturated salt solution, (iv.) petroleum, (v.) methylated spirit, (vi.) glycerine. After each observation the hydrometer *must be well rinsed and wiped dry.*

CALCULATIONS.

8. A ship is said to draw more water in a river than at sea. If this be so, what is the reason?

9. A body whose volume is 5 cub. ft. and specific gravity 1·2 hangs by a wire suspended in water, which completely covers it. What force exerted along the wire is needed to support the body?

10. A body whose specific gravity is 1·4 and volume 3 cub. ft. is placed in a vessel in which there is water enough to cover it. What thrust does the body produce on the bottom at the points at which it is supported?

11. A ball of beechwood (sp. gr. ·7) weighs 3 lbs.; it is fastened by a thread to the bottom of a barrel; the length of the thread and the diameter of the ball are together less than the height of the barrel. If the barrel be filled with water, what position will the ball take and what will be the tension of the thread?

12. A cube of wood whose edge is 10 ins. and specific gravity ·6 floats in water. What weight must be placed on it in order to just totally immerse it?

13. A man weighing 160 lbs. floats with 4 cub. ins. of his body above the surface. What is his volume in cubic feet?

14. A piece of silver, whose specific gravity is 10·5, weighs 120 oz. in air. How much does it weigh in water?

15. A piece of copper weighs 10 lbs. in air and $8\frac{3}{4}$ lbs. in water. Find its specific gravity and its volume in cubic inches.

16. The specific gravity of a substance is 7·5; a portion of it weighs 390 grains in water. What is its weight in air?

17. If 1 c.c. of metal weighs 8·5 gms. in water, what is its real weight?

18. A glass ball weighs 1000 grains; it weighs 630 grains in water, and 650 grains in wine. Find the specific gravity of the wine.

19. A piece of glass weighs 47 gms. in air, 22 in water, and 25·8 in alcohol. Find the specific gravity of the alcohol.

20. Find the specific gravity of a liquid in which a hydrometer weighing 20 gms. floats with 18 cub. cms. of its volume immersed.

21. A hydrometer floats in water with $\frac{4}{5}$ of its volume immersed, and in another liquid with $\frac{5}{6}$ of its volume immersed. Find the specific gravity of the other liquid.

CHAPTER VII.

FLUID PRESSURE.

56. Fluid Pressure. — Any fluid is, under ordinary circumstances, in a state of pressure throughout its whole volume. That is to say, the various parts of the fluid are pressing against one another, and against the sides of the vessel containing the fluid. Frequently the pressure is different at different parts of the fluid.

These pressures are most conveniently measured in terms of *force per unit area.* Thus we may have a pressure expressed in pounds weight per square foot or square inch, or grammes weight per square centimetre.

The pressure of the atmosphere on all surfaces exposed to it is about 15 lbs.-wt. per square inch. The pressure in water at a depth of 35 ft. is about 30 lbs.-wt. per square inch; 15 lbs.-wt. due to the atmosphere pressing on the top and 15 lbs.-wt. due to the water itself.

57. Fundamental Properties of a Fluid. — A fluid, in virtue of its mobility, possesses two fundamental properties in relation to fluid pressure. These are—

(i.) The pressure exerted on any surface by a fluid at rest is everywhere perpendicular to that surface.

(ii.) If any extra pressure per unit area is impressed on a fluid, this extra pressure is transmitted without change throughout the liquid and is exerted everywhere on the bounding surface of the liquid.

This second property is the **Principle of Transmission of Fluid Pressure,** and is sometimes known as **Pascal's Law.**

58. Experimental Verification of Pascal's Law.—Let a closed vessel of any shape be filled with water, or other fluid (Fig. 41). Let short tubes of equal sectional area (say, 1 sq. in.) be attached to openings made in different parts of the walls of the vessel, and let these tubes be closed with tight-fitting pistons (working with as little friction as possible), acted upon by such forces as are required to keep them in place, *i.e.* to balance the liquid pressures. If, now, an *additional* force, say, of 1 lb., be applied to any one of the plugs (say A), it will be found necessary to apply an *additional* force of 1 lb. to each of the other plugs B, C, D, to prevent their coming out; similarly, if the force on A be increased by 2 lbs. or any other amount, the force applied to each of the other plugs will also have to be increased by the same amount. Hence an extra pressure of 1, 2, or more pounds per square inch imparted to the surface at A gives rise to an equal extra pressure over every other square inch of the surface.

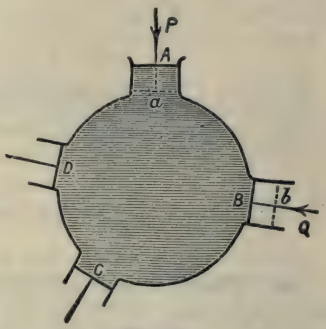


Fig. 41.

59. Pressure at a Point in a Fluid.—If we consider the force on a *very small* area round a particular point in a fluid, and calculate the *average force per unit area* on this small area, we get approximately the *pressure at this point* in the fluid.* The smaller the area considered the more exact is the approximation.

For instance, if the area measures $\frac{1}{100}$ sq. inch, and the total thrust on this area is ·3 lb., then the pressure at this point is at the rate of 30 *lbs. per sq. inch.*

The pressure at any point in a fluid is the same in all directions—that is to say, the pressure on any given *small* area containing the given point is the same *in magnitude* in

* This is really the *definition* of the term "pressure at a point."

whatever direction the area is supposed to face; though the *direction* of this pressure will of course change if the area is turned into a new position, for the pressure is always perpendicular to the area.

This is a difficult theorem to prove, and the student must be content to assume its truth.

60. Pressure at any Point in a Liquid at Rest. — The pressure at any point in a heavy liquid at rest depends upon the depth of the point below the free surface of the liquid. For, if we consider a horizontal unit area placed at a given depth in a liquid, it is obvious that the force upon this area will be vertical, and will be equal to the weight of the column of liquid directly above the area together with the weight of the column of air vertically above this area.

Thus the force F on an area A at a depth h in a liquid of density d is equal to the weight of a column of liquid of area of cross-section A of height h together with the force f exerted on an area A of the surface by the atmosphere.

$$\therefore \quad F = Ahd + f.$$

Now **force per unit area is called the pressure,** hence, if P denote the (total) pressure at the given point, and p the atmospheric pressure on the surface, we have

$$F = PA, \quad f = pA,$$

$$\therefore \quad PA = Ahd + pA,$$

hence

$$P = hd + p,$$

which gives the pressure at an immersed point in terms of the atmospheric pressure, the depth to which it is immersed, and the density of the liquid.

If A is in sq. ins., h in ins., and d in pounds per cub. in., then F and f must be expressed in pounds-weight, and P and p in pounds-weight per sq. in.

If A is in sq. cms., h in cms., and d in grammes per cub. cm., then F and f must be expressed in grammes-weight and P and p in grammes-weight per sq. cm.

Note that to calculate the pressure at any depth we calculate the *vertical* pressure. By § 59 the pressure at this depth in any other direction is equal to this vertical pressure.

61. Surfaces of Equal Pressure in a Liquid at Rest. — In a liquid at rest the pressure at all points in a *horizontal surface* is the same—that is, the surfaces of equal pressure in a liquid at rest are horizontal.

Consider two points, A and B, in the same horizontal plane. If the pressure at A is greater than at B, the liquid cannot be at rest in the plane, but must flow from A to B; for, *since gravity acts vertically*, the *weight* of the liquid cannot cause or prevent its motion in a horizontal plane, and the liquid can therefore be at rest only when there is no difference in pressure between points in the same horizontal plane.

62. The Free Surface of a Liquid at Rest must be Horizontal.—The free surface of a liquid is the surface exposed to the external atmosphere. The pressure at all points on this surface is the same—the pressure of the atmosphere—and, by the preceding paragraph, the surface must therefore be horizontal.

63. Water finds its own Level.— The preceding arguments are rough explanations rather than mathematical proofs, and it is beyond the purpose of this elementary book to carry these explanations any farther. Before leaving the subject, however, two other important theorems must be stated. They are easy to understand, though they are difficult to prove.

(i.) *Water always finds its own level.*—When a liquid is at rest in a vessel the free surface is always horizontal. This is true whether the liquid is at rest in a vessel like a tank with an unbroken free surface or in a vessel *with communicating parts*, such as that shown in Fig. 42, where the free surface is broken up into parts at A, B, and C. The three surfaces at A, B, and C are in the same hori-

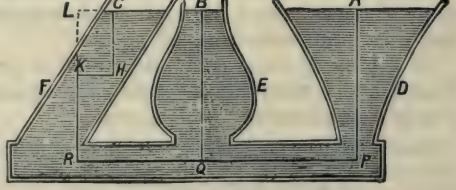


Fig. 42.

zontal plane. In this vessel, if water be poured into the part D until it reaches a certain level in this part, *it will rise to or "find" the same level* in the parts E and F. In the same way, if two lakes communicate with each other by a subterranean passage *full of water*, the level of the water in the two lakes will be the same.

Another illustration of this principle is found in the water-supply system of a town. The reservoir and the network of pipes communicating with it constitute, when all taps and valves are open, one large vessel with many parts. The water from the reservoir fills all the parts and tends to rise in every pipe leading to the open air to the level it has in the reservoir. Hence, if the reservoir is higher than the highest point to which water has to be supplied, water will always flow from an open tap attached to the system of pipes.

(ii.) *The pressure at any point depends on the depth of the point below the free surface*, whether the point is directly below the free surface or not.

Thus, in Fig. 42, the pressure at K exceeds the pressure of the atmosphere by the weight of a column of liquid of height KL or HC and of unit cross-section, and the pressure at R exceeds the atmospheric pressure by the weight of a similar column of liquid of height RL. RL is called the "head" of liquid at the point R.

Again, if we return to the water-supply system of a town, the greater the height of the water level in the reservoir above a given tap the greater will be the pressure of the water at this tap. For example, if the level in the reservoir or supply cistern is 50 ft. above the tap, then the tap is really at a depth of 50 ft. below the free surface of the water, and the pressure in the pipes at the point where the tap is placed is that due to a *head* of 50 ft. of water.

In this connexion we need take no account of the transmitted atmospheric pressure, as this is neutralized by the actual air pressure at the mouth of the tap. The flow of water would be much more rapid if the tap opened into a room containing no air.

The above rule is not, however, accurately true, being modified by the fact that there is considerable friction between the water and the pipes; this tends to diminish the pressure with which the water is forced from a tap.

64. The U-tube.—It is easily shown by experiment that when two immiscible* liquids of different densities are poured into a vessel the liquids arrange themselves so that the heavier goes to the bottom and the lighter to the top. If, however, we take a **U**-tube—which, as its name implies, is simply a glass tube bent into the form of an elongated **U**—and pour at first a heavy liquid down one arm and then a lighter liquid down the other arm: we shall find as a rule that the heavier liquid occupies the bend of the tube, and that the liquids arrange themselves as in Fig. 43, where the free surface (P) of the heavier liquid is at a lower level than the free surface (R) of the lighter liquid, and at a higher level than the common surface (Q) of the two liquids.

Fig. 43.

In § 61 it was stated that the pressure at all points in a horizontal surface in a liquid at rest is the same. Therefore, if we take an imaginary section at O on a level with the surface of separation Q, we have the pressure at O due to the head of liquid PO is equal to the pressure at Q due to the head of liquid QR. If h_1, h_2, d_1, d_2 be the heights and densities, respectively, of the two liquids in OP and RQ and P the atmospheric pressure, we have (§ 60)

$$h_1d_1+p = h_2d_2+p.$$

$$h_1d_1 = h_2d_2.$$

$$\therefore \quad \frac{h_1}{h_2} = \frac{d_2}{d_1} = \frac{s_2}{s_1},$$

where s_1, s_2 are the specific gravities of the two liquids That is, **the heights of the free surfaces above the common surface are inversely proportional to the specific gravities of the liquids.**

Thus, by means of the **U**-tube, the specific gravities of liquids which do not mix can be readily compared and found without using the balance. The heights of the free surfaces P, R above the surface of separation Q can be measured by a scale of inches or millimetres placed behind the two tubes as in Figs. 43, 44.

* *I.e.* liquids which will not mix.

Exp. 29.—*Find the specific gravity of Mercury.*—Take a piece of glass tubing about a yard long and $\frac{3}{8}$ in. bore. Heat the middle gently in a bats-wing burner until it softens, and then bend it to form a U-tube, bringing the long arms as close together as you can (Fig. 44). Fix the tube vertically in a clamp with the bend resting on the table. Pour mercury down one arm until there are about 2 ins. of mercury in each limb. Then pour water in one arm until the column of water is about 6 ins. in height. Measure the heights of the mercury and water surfaces from the surface of separation by a millimetre scale. Pour in more water and repeat the measurements. Repeat the addition of water several times until one side of the tube is full of water. Tabulate your results thus :

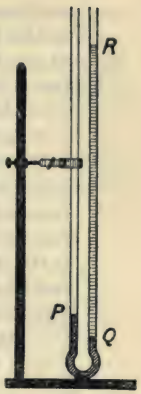


Fig. 44.

Height of Mercury.	Height of Water.	Ratio.

The last column should be constant and equal to 13·6.

Note, however, that the readings when the heights are large give more accurate results than when the heights are small. This is because on a large distance an error of 1 mm. gives a much smaller percentage error than on a small distance.

Repeat the experiment with various immiscible liquids which are available, and compare your results with those obtained by the bottle and the weighing of solids.

If the liquids are miscible, an inverted U-tube (**Hare's apparatus**) may be used. When suction is applied at the open end of a tube dipping into water, water rises in the tube, being pushed upwards by the pressure of the atmosphere. The magnitude of this pressure due to the atmosphere will be considered later.

Exp. 30.—*Find the Specific Gravity of a saturated solution of Copper Sulphate.*—Place the limbs of the U-tube (Fig. 45) one in a beaker containing water, the other in a beaker containing the

solution. Clamp the tube so that it is vertical. Suck at the open end of the T piece until the level of the lighter liquid is within a few cm. from the top. Then clamp the rubber tubing so that it is air-tight. (How can this be tested?)

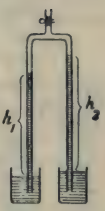


Fig. 45.

Measure the height of each level above the level in the beaker beneath it by a mm. scale. Let h_1, h_2 cm. be the heights and d_1, d_2 the corresponding densities of the solution and water respectively. The pressure of the air in the tube is the same above each column of liquid; call it P, and let H be the pressure of the atmosphere on the liquids in the beakers.

Then $H = P +$ pressure due to column h_1.

Also $H = P +$ pressure due to column h_2;

$\therefore$ pressure due to h_1 = pressure due to h_2.

$$\therefore \quad h_1 d_1 = h_2 d_2 ;$$

$$\therefore \quad \frac{h_2}{h_1} = \frac{d_1}{d_2} = \frac{s_1}{s_2},$$

but $s_2 = 1$; $\quad \therefore \quad s_1 = \frac{h_2}{h_1}$.

Make several determinations, taking care to read afresh from the level in the beaker each time. Enter your results as in Exp. 29. (Why is it unnecessary to know the cross sections of the tube?)

65. Pressure of the Atmosphere.—Since air has weight, the atmosphere must exert a pressure on all surfaces with which it is in contact.

The effect of atmospheric pressure may be illustrated by the following experiment:—

Exp. 31.—Take a glass tumbler filled to the brim with water and lay a sheet of cardboard over the top, pressing it well down. will be found that the glass may be inverted without the water falling out (Fig 46). The card is in fact held up by the upward thrust of the atmosphere on its under side.

Fig. 46.

This upward thrust has to support the weight of the card and the thrust of the water on the upper side, besides pressing the card tightly against the rim of the glass.

Hence the *pressure* of the air acting upwards on the card must exceed the pressure of the water downwards; otherwise the card would fall down. The atmospheric pressure is therefore greater than the pressure due to a column of water of the same height as the glass.

66. Torricelli's Experiment.—The Barometer.—The first actual measurement of the pressure of the atmosphere is due to Torricelli (1643), and his experiment resulted in the invention of the mercurial barometer.

Exp. 32.—To perform the experiment or to construct a barometer in its simplest form, a glass tube (Fig. 47) about 33 ins. long and closed at one end is completely filled with mercury. The open end is then closed with the finger, the tube inverted into a cup of mercury, and the finger then removed, care being taken not to allow any air to get into the tube. The mercury will at once sink and leave a clear space at the top of the tube and the height of the column of mercury above the surface in the cup will be found to be about **30 inches** or **760 millimetres.**

Fig. 47

If the tube be furnished with a scale for reading off the height of the mercury, the apparatus constitutes a **mercurial barometer.**

The space above the mercury is practically a vacuum, and is called the **Torricellian vacuum.*** Hence there is no pressure at the top of the tube.

The explanation of this experiment is as follows:—The surface of the mercury in the cup is exposed to the pressure of the atmosphere; but the surface of the mercury *in the tube* is not exposed to any pressure. We should therefore expect that the mercury would stand at a higher level inside the tube than outside; for, if by any means we could lower the surface of the mercury within the tube to the same point as that outside the tube, the atmospheric pressure on the surface outside the tube, being balanced by no corresponding pressure within, would tend to force the mercury up the tube again.

* Strictly, it contains a very minute quantity of the vapour of mercury; see § 71.

Exp. 33 — *To show that the height of the barometer does not depend on the shape, size, or slope of the tube.*—Repeat **Exp. 32**, with different tubes of different shape and size, and holding them at all slopes. In each case measure the perpendicular height of the surface at P (Fig. 47) above the surface in the cistern Q. It will be the same in each case.

By applying the formula of § 60 we see that, if h is the height of the barometer when a liquid of density d is used, the atmospheric pressure is given by

$$P = hd\,;$$

hence we obtain the rules—

If the height of the mercury barometer is h ins., the atmospheric pressure in pounds-weight per sq. in. = the weight in pounds of h cub. ins. of mercury.

If the height of the mercury barometer is h cms., the atmospheric pressure in grammes-weight per sq. cm. = the weight in grammes of h cub. cms. of mercury.

The normal height of the mercury barometer is 30 ins. or 76 cms.; hence the atmospheric pressure is

$$\frac{30\times1\times62\frac{1}{2}\times13{\cdot}6}{1728} = 14\tfrac{3}{4} \text{ lbs. wt. per sq. in.}$$

or $$76\times1\times1\times13{\cdot}6 = 1034 \text{ gms. wt. per sq. cm.}$$

[N.B.—It is useful to remember the above numbers.]

67. Water and Glycerine Barometers. — Instead of performing Torricelli's experiment with mercury, we might use a column of water, glycerine, or any other liquid to measure the pressure of the atmosphere, provided that we took a sufficiently long tube for the purpose.

Example.—When the mercury in a mercury barometer stands at 30 ins., find the height of the water barometer.

The density of mercury is 13·6 times that of water.

$\therefore$ pressure due to 1 ft. of mercury = pressure due to 13·6 ft. of water;

$\therefore$,, ,, $2\frac{1}{2}$,, ,, = ,, ,, $(13{\cdot}6\times2\frac{1}{2})$,,

$\therefore$ height of water barometer = $(13{\cdot}6\times2\frac{1}{2})$ ft.

= 34 ft.

Unless, therefore, the tube exceeded 34 ft. in height, no vacuum would be formed and the instrument would be useless.

68. The **water-barometer** is much more sensitive to small changes of atmospheric pressure than a mercurial barometer.

For the column of water is *always* 13·6 times as high as the column of mercury. Thus the change of pressure which would cause the mercury to rise ·1 in. would cause the water to rise 13·6 times as much, or 1·36 ins.

The great objection to a water barometer is the difficulty of securing a good vacuum at the top of the tube. Not only does water evaporate freely into the vacant space, but air is absorbed at the surface of the cup, and is given off again at the surface of the column.

These objections are to a great extent obviated by the use of glycerine. Its specific gravity being 1·26, the **glycerine barometer** is more than ten times as sensitive as a mercurial barometer, and a much better vacuum is obtained than with water.

Example.—When the water barometer is at a height of 34 ft., find the height of a glycerine barometer, the specific gravity of glycerine being 1·26.

The press. due to 1·26 ft. of water = press. due to 1 ft. of glycerine ;

$\therefore$,, ,, 34 ,, ,, = ,, ,, (34 ÷ 1·26) ,,

$\therefore$ height of glycerine barometer = (34 ÷ 1·26) = 27 ft. nearly.

69. The Siphon Barometer consists of a **U**-tube (Fig. 48) which has branches of unequal length. The shorter branch is open to the atmosphere and corresponds to the cup of Torricelli's instrument, while the longer one is closed, and a vacuum is formed above the mercury at its upper end. When the mercury rises in one arm it falls in the other, and the height of the barometer is the difference of level of the mercury in the two branches. It is often read off on a graduated dial by means of the arrangement shown in Fig. 49 (Wheel Barometer).

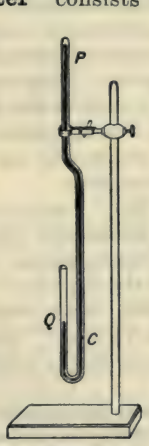


Fig. 48.

Fig. 49.

70. The use of the Barometer is to indicate the Pressure of the Atmosphere.—The average height of the mercurial barometer is generally taken as **30 ins.** or **760 mm.** This corresponds to **34 ft.** height of the *water barometer*, or an *atmospheric pressure* of about **14·7 lbs. per sq. in.** This pressure is called **one atmosphere.** But we know from common experience that the barometer is constantly rising or falling, indicating that the atmospheric pressure fluctuates considerably within certain limits; moreover, it varies at different altitudes above sea level.

If the barometer rises, it indicates an increase in the atmospheric pressure, while a falling barometer indicates a decrease of pressure.

The reason why the barometer can be used to predict the weather is because experience has shown that certain changes of weather are generally accompanied by certain changes of atmospheric pressure.

Thus, when we say that "the barometer usually falls for rain," we mean that rainy weather is usually preceded by a decrease in the pressure of the atmosphere. Similarly, an improvement in the weather usually occurs simultaneously with an increase of pressure. The same changes are not indicated in the same manner in all parts of the globe. But in no case can changes of weather affect the height of the barometer otherwise than by causing changes in the pressure of the atmosphere.

71. The Torricellian Vacuum. — We have already pointed out that the space above the mercury in a barometer tube is not a perfect vacuum, but contains a minute quantity of mercury vapour. This vapour exerts a very small pressure on the mercury, so that the column is not so high as it would otherwise be. The difference is so small that it may usually be neglected, amounting as it does to about $\frac{1}{1000}$ mm. at ordinary temperatures.

In all cases the "vacuum-space" contains vapour of the liquid in the barometer, the quantity of vapour per unit volume of vacuum space varying with the nature and temperature of the liquid.

The barometer may also be *faulty* in having a small quantity of air in the "vacuum-space," in which case a considerable error in the reading may occur.

72. The **Aneroid Barometer** is a hollow metal box exhausted of air. The atmospheric pressure tends to force in the top of the box, but is resisted by the elasticity of the metal, which acts like a spring.

When the pressure increases or decreases the lid sinks or rises slightly and moves a pointer which indicates the pressure on a dial. This dial is graduated in "inches" or "millimetres" corresponding to the readings of a mercurial barometer. The aneroid is chiefly used on account of its portability.

73. Pumps.—The atmospheric pressure may, by means of a suitable mechanical arrangement, be made to raise liquid. Such an arrangement is called a **pump** (Fig. 50). It consists essentially of a cylinder and piston, as in the common syringe. In order that the action may be continuous, the cylinder and the piston are each fitted with a valve opening upwards. As the piston is raised, the atmospheric pressure closes its valve and opens that of the cylinder, liquid being at the same time forced upwards towards the cylinder. When the piston descends, the pressure of the air below it opens its valve and closes that of the cylinder.

Fig. 50.

In the **Force pump** (Fig. 51) the piston has no valve, but the lower end of the cylinder communicates with a pipe closed by a valve. When the liquid has been raised to the cylinder, the piston, on its downward stroke, closes the lower valve and forces liquid up the pipe. Since the atmospheric pressure cannot balance a column of water more than 34 feet high, these pumps can be used for water only when the distance of the cylinders from the original level of the water is less than 34 feet.

Fig. 51.

74. Air Pumps.—Pumps for the removal of gases are made on the same principle as those used for lifting liquids. They consist of a cylinder and piston fitted with valves that act in just the same way as those of a water pump. The vessel from which the air is to be exhausted is called the **receiver.** If the valves are made to open in the opposite direction, then air is forced into the receiver instead of being drawn out. A bicycle pump acts on this principle.

Instead of using a cylinder and piston, air can be exhausted from a vessel by means of a liquid dropping down a fine tube, to which the receiver is connected by a side tube. As each drop of liquid passes the side tube it carries before it a small bubble of the air, which it sweeps out at the bottom.

In the *Sprengel pump* (Fig. 52) mercury is the liquid used. The pump acts slowly, but the exhaustion is very thorough and the labour small, since the action can be made automatic.

In the filter pump (Fig. 53), used largely in the laboratory, water under pressure is used to remove the air. Its action is similar to that of the mercury in a Sprengel pump.

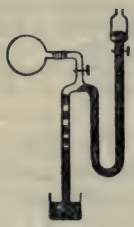


Fig. 52.

Fig. 53.

75. Boyle's Law.—In § 3 it was stated that, although the volumes of solids and liquids do not depend upon the pressure exerted on them, the volumes of gases do greatly depend upon the pressure. When the pressure on a gas is increased the volume is diminished; when the pressure is decreased the volume is increased. The relation between the pressure and the volume of a gas at a constant temperature was discovered by Robert Boyle, of Lismore, Ireland, in 1662. It may be thus stated:

Boyle's Law.—The volume of a given mass of gas kept at constant temperature is inversely proportional to the pressure on the gas.

Thus, let p be the pressure of a gas occupying the volume v.

Then at pressure $2p$ the volume of the gas will be $\frac{1}{2}v$,

" $3p$ " " " $\frac{1}{3}v$,

" $\frac{1}{2}p$ " " " $2v$,

" p/n " " " nv,

and so on. Since

$$pv = 2p \times \tfrac{1}{2}v = 3p \times \tfrac{1}{3}v = \tfrac{1}{2}p \times 2v = p/n \times nv,$$

it follows that

The product of the pressure into the volume of a given mass of gas at constant temperature is constant.

For let v_1 be the volume of the gas when the pressure is p_1, v_2 its volume when the pressure is p_2, the temperature being the same in both cases. Boyle's Law states that p_1, p_2 are inversely proportional to v_1, v_2; that is

$$\frac{p_1}{p_2} = \frac{1}{v_1} \Big/ \frac{1}{v_2} = \frac{v_2}{v_1},$$

or, clearing of fractions, we may state Boyle's Law in symbols thus:

$$p_1v_1 = p_2v_2 = \ldots = \text{a constant}$$

for the same mass of gas at the same temperature.

Exp. 34.—*To verify Boyle's Law experimentally in the case of air for pressures greater than that of the atmosphere.*—A piece of apparatus called **Boyle's tube** is generally used (Fig. 54). To make it, take a long piece (say 5 ft.) of stout glass tubing of about $\frac{1}{4}$ in. bore; clean and dry it and then carefully heat one end and seal it. At about 10 ins. from this end heat the tube and bend it through two right angles. Mount on a board and attach mm. scales to each limb, as in the figure, having the zeros at the same level and as low as possible. Insert a small funnel into the open end for the purpose of pouring in mercury. Take the apparatus into a corner of the room where the temperature will remain nearly constant. Pour a little mercury into the bend and adjust by shaking until the mercury surfaces are both at zero (Fig. 54). A certain mass of air has now been imprisoned in the short limb at atmospheric pressure. Pour a little mercury into the long limb. Note that the mercury does not rise in the closed limb as rapidly as in the open limb: this is due to the enclosed air exerting pressure. Read the two levels. Pour in more mercury and repeat the readings. Proceed till the long tube is full. Now read the barometer and note its height. The difference in level of the two surfaces in the Boyle's tube plus the barometric height is the total pressure exerted on the gas, and therefore exerted by the gas. Thus in Fig. 55 the pressure on the gas in AP over and above the atmo-

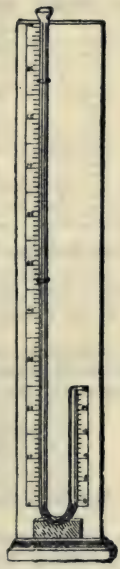


Fig. 54.

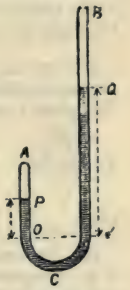


Fig. 55.

spheric pressure is equal to that of a column of mercury of height $O'Q - OP$, and therefore, if the barometric height is H, the total pressure on the gas is $H + O'Q - OP$. The volume in AP of the gas is found by subtracting the reading of the mercury surface in the closed limb from the reading of the top of the tube. Enter the results thus :

Ht. of Mercury in closed limb. I.	Ht. of Mercury in open limb. II.	Excess. Press. over Atmospheric = I. − II. III.	Total Press. = Barometric Ht. + III. IV.	Vol. of Air deduced from I. V.	Product, Pressure × Volume. VI.
cms.	cms.	cms.	cms.		
0	0	0	76	25	1900
5	24	19	95	20	1900
10	61	51	127	15	1905
...	...	...	...	...	...

The products in Column VI. are practically constant : thus, Boyle's Law is verified.

To verify Boyle's Law for any gas it is only necessary to substitute that gas for air in the closed limb.

Examples.—(1) *Suppose* 10 *c.ft. of oxygen at atmospheric pressure* (15 *lbs. per square inch*) *to be forced into a steel cylinder of* 0·6 *c.ft. capacity. Required the pressure of the gas in pounds-weight per square inch.*

Apply $p_1 v_1 = p_2 v_2$;

we obtain $15 \times 10 = p_2 \times 0{\cdot}6$;

$\therefore \quad p_2 = 250$,

i.e. the final pressure = 250 lbs.-wt. per square inch.

(2) *Determine the barometric height when a milligramme of air at* 0° *C. occupies a volume of* 19·0 *c.c. in a tube over mercury, the mercury standing* 73·5 *cms. higher inside the tube than outside* (1 *gramme of air at* 0° *C. under a pressure of* 76 *cms. of mercury occupies* 773·3 *c.c.*).

If H = the barometric height, the pressure, p_1, of the air in the given tube is $(H - 73{\cdot}5)$ cms. of mercury and $v_1 = 19{\cdot}0$. Also, for the same mass of air, when $p_2 = 76$, $v_2 = {\cdot}7733$.

Therefore, applying Boyle's Law, we have

$$(H - 73{\cdot}5) \times 19{\cdot}0 = 76 \times {\cdot}7733,$$

or
$$H = \frac{76 \times {\cdot}7733}{19{\cdot}0} + 73{\cdot}5 = 3{\cdot}1 + 73{\cdot}5$$

$$= 76{\cdot}6 \text{ cms. of mercury.}$$

Summary.—Chapter VII.

1. Pressure = force per unit area. (§ 56.)

2. Fluid pressure on a surface is everywhere perpendicular to the surface. (§ 57.)

3. Pressure impressed on a fluid is transmitted throughout the fluid (Pascal's Law). (§ 57.)

4. Pressure at a point in a fluid is the average force per unit area for a *very small* area which includes the point. (§ 59.)

5. The pressure at a depth h in a heavy liquid differs from the pressure at the surface by the weight of a column of the liquid having a cross section of unit area and a height h. (§ 60.)

6. In a liquid at rest the pressure is the same at all points in a horizontal surface. The free surface is therefore horizontal. (§§ 61–62.)

7. The pressure at any point which is not directly below the free surface depends on its vertical depth below the level of that surface. (§ 63.)

8. If a U-tube contains two liquids, the heights above the common surface are inversely proportional to the densities, *i.e.*,

$$\frac{h_1}{h_2} = \frac{d_2}{d_1}. \qquad (\S\ 64.)$$

9. The height of the barometer measures the pressure of the atmosphere. (§ 66.)

10. The average height of the mercury barometer
= 30 ins. = 760 mms. (§ 66.)
∴ the average height of the water barometer
= 30 ins. × 13·6 = 34 ft. (§ 67.)
∴ average pressure of atmosphere
= 14·7 lbs. wt. per sq. in.
= 1033 gms. wt. per sq. cm. (§ 70.)

11. The principal forms of the barometer are—
(1) The common barometer, having cup of mercury. (§ 66.)
(2) The siphon, or bent tube barometer. (§ 69.)
(3) The aneroid barometer (not mercurial). (§ 72.)

12. *Boyle's Law.*—The volume of any given mass of gas varies inversely as its pressure, temperature remaining constant, *i.e.*

$$pv = \text{a constant or } p_1v_1 = p_2v_2, \text{ \&c.} \qquad (\S\ 75.)$$

EXERCISES X.

Calculations.

1. How is fluid pressure measured when uniform? Compare the pressures of 15 lbs. per sq. in. and of 1,000 oz. per sq. ft.

2. The neck and bottom of a bottle are $\frac{1}{2}$ in. and 4 ins. in diameter respectively. If, when the bottle is full of water, the cork is pressed in with a force of 1 lb., what extra force is exerted upon the bottom of the bottle?

3. Two communicating cylinders (containing water), the diameters of whose bases are 3 ins. and 8 ins. respectively, are fitted with pistons. If a weight of 27 lbs. be placed on the smaller piston, what weight must be placed on the larger to keep it at rest?

4. Find in pounds per square inch the pressure in water at a depth of 32 ft.

5. The pressure at a point 3 ft. below the surface of a heavy fluid is 30 lbs. per square inch, and at a depth of 7 ft. it is 50 lbs. What is the pressure at the surface?

6. If the atmospheric pressure at the surface of the Earth be $14\frac{1}{2}$ lbs. per square inch, find the theoretical height of the water barometer, *i.e.* a barometer containing water instead of mercury.

7. The glycerine barometer is found to stand at 329·2 ins. when the mercurial barometer stands at 30·61 ins. Given that the specific gravity of mercury is 13·569, find the specific gravity of glycerine.

State the physical principles that justify your calculation.

8. In a tube of uniform bore a quantity of air is enclosed. What will be the length of this column of air under a pressure of 3 atmospheres, and what under a pressure of $\frac{1}{3}$ atmosphere, its length under the pressure of a single atmosphere being 12 ins.?

9. Mercury is poured into a uniform bent tube, open at both ends, and having its two branches vertical. One end is closed, its height above the mercury being 4 ins. How much mercury must be poured into the open end so that the mercury may rise 1 in. in the closed branch?

10. A wide-mouthed bottle full of air is closed with a well-ground glass stopper, 5 cms. in diameter, when the barometer stands at 772 mms. What weight must the stopper have in order that it may be just blown out if the barometer goes down to 730, the temperature remaining the same?

11. The following readings were taken with a Hare's apparatus:—

	cms.	cms.	cms.	cms.
Height of water column	69·00	66·85	63·60	61·20
Height of column of a salt solution	61·85	59·90	57·05	55·00

Find the specific gravity of the solution.

CHAPTER VIII.

CO-ORDINATES AND CURVE-PAPER.

76. The Use of Squared Paper.—Squared paper has already been found very useful in the determination of areas. It is even more useful when we wish to show how one quantity depends on another. When experiments are made to find out how one quantity varies with another quantity, the results of the experiments are first of all noted in terms of two units, and thus are expressed in figures. For example, if we wish to find out something about the rate of evaporation of water in air, we can start with a known weight of water in a dish, and, leaving it exposed to the air, weigh it at noon each day until it has become completely vaporised. Here we are measuring two things, (1) the amount of water that remains, (2) the time that has elapsed since the commencement of evaporation.

The results can be set down in two columns, thus:

Time of Exposure.	*Weight of Water left.*
0	50·0 grammes
1 day	44·2 ,,
2 days	40·1 ,,
3 ,,	35·5 ,,
4 ,,	30·2 ,,
5 ,,	23·9 ,,
6 ,,	18·0 ,,

On looking at these figures we at once obtain an idea of the rate of evaporation, but a far clearer idea is obtained if the results are plotted graphically on squared paper; for then the relation between the two sets of quantities can be represented (together) by a single line.

To plot the results, two lines (called *Axes*) are first drawn at right angles to each other, and near the edges of the squared paper. Along the horizontal axis, *OA* (Fig. 56), the time of exposure to evaporation is measured, each day being represented by a certain number, say eight, of the

small divisions. Along the vertical axis, OB, the weight of water is measured, 2 grammes being represented by a certain number, say one, of the small divisions.

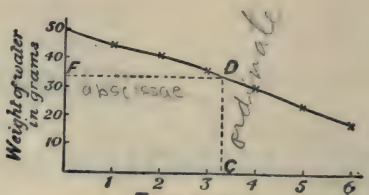


Fig. 56.

The point where the vertical line through the 1-day division on the horizontal axis cuts the horizontal line through the 44·2-gramme mark is noted, and a small × made there. The point of intersection of the perpendiculars through the 2-day and the 40·1-gramme divisions is marked in the same way, and so for all the points. The points are then joined by a freehand curve which passes through them all.

In this way the variation of the weight of water with the time of exposure is represented by a single line. The slope of the curve at any point indicates the rate of evaporation there; where the curve is most steep the weight of water lost per day is greater at that time than at any other time.

The points might have been joined by straight lines. Had this been done, the zigzag line obtained would have implied that there was a sudden change in the rate of evaporation at noon each day. Now the rate of evaporation depends chiefly on the dryness of the air. This would not change suddenly at noon, and therefore such a zigzag would not have represented the facts truly. The change was quite gradual, and therefore it is properly represented by a smooth curve.

77. Use of the Curve.—From the curve of Fig. 56 the weight of water at any time during the evaporation can be found, although actually the water was weighed only at noon each day. Suppose we wish to know the weight at 6 P.M. on the third day. This time is represented by the point C on the horizontal axis. The vertical through C cuts the curve at D, and the horizontal line through D cuts the axis of weight at a point F, which represents 33·5 grammes. Hence at 6 P.M. on the third day the weight of water was 33·5 grammes.

This method of finding values lying between those actually obtained by experiment is called *Interpolation.* If the curve is regular, values lying beyond the last one found by experiment can be found in the same way. The method is then called *Extrapolation.*

78. Definitions.—The point of intersection of the axes is called the *Origin.* The perpendiculars let fall from any point to the axes are called the *Co-ordinates* of that point. Vertical distances, such as *CD*, are called *Ordinates.* Horizontal distances, such as *DF*, are called *Abscissae.* Since the curve illustrates the relation that holds between two quantities, it follows that the lines representing those quantities have a certain relation to each other. Thus the ordinate (y) of any point on the curve is related in some way to the abscissa (x) of the same point, and this relation can sometimes be easily expressed by an equation. For instance, when a straight line passes through the origin and bisects the angle between the axes, the ordinate of any point is clearly equal to the abscissa of the same point. This fact is expressed by the equation $x = y$.

Summary.—Chapter VIII.

1. Squared paper, sometimes called curve-paper, is very useful for plotting curves between two sets of variables. (§ 76.)

2. The position of a point on the curve is measured from two **axes** drawn, from an **origin,** at right angles to each other. These distances are called **co-ordinates.** The co-ordinate measured from the axes drawn across the paper is called the **ordinate;** that from the axes drawn up and down the paper the **abscissa.** (§ 77.)

3. When plotting points from a set of experimental numbers first draw the two axes and then mark off along them suitable scales for the representation of the points. Then plot the points from the given data and finally join the points by means of a smooth curve or a straight line. (§ 76.)

EXERCISES XI.

PRACTICAL.

1. Draw a line through the origin such that, if any point on it be taken, the distance of that point from the horizontal (X) axis is twice its distance from the vertical (Y) axis. What is the equation of the line?

2. Draw a line such that its distance from the vertical axis is always three times its distance from the horizontal axis. What is its equation?

3. Draw a circle of 3 cm. radius, with its centre at the origin, and find its equation; *i.e.* find the relation that holds between the radius and the co-ordinates of any point on the circumference.

4. Draw axes of reference and plot the points whose abscissae and ordinates are respectively 0, 0; 2, 1; 3, 2; 4, 4; 5, 7; and draw a curve through them.

5. Plot the points whose abscissae and ordinates are respectively
2, 0; 3, 1; 4, 2; 5, 4; 6, 4.
Draw a curve through them. What sort of a curve is it?

6. Draw the curves passing through the two series of points I., II.

Mark the point where the curves intersect, and state what you consider to be the values of the co-ordinates of that point.

I.

Abscissa.	1	2	3	4	8
Ordinate.	0	2	3	3·6	4·4

II.

Abscissa.	0	1	2	4
Ordinate.	4	2	1	0

7. Through the origin draw a line that shows the relation between inches and centimetres.

8. Find the areas of circles whose radii are 1, 2, 3, 4, and 5 cm.; plot these areas along one axis, and the corresponding radii along the other axis. Draw a smooth curve through the points obtained. This curve is called a *parabola*. Clearly its equation is $y = 4\pi x^2$.

Suppose the radii are plotted as abscissae, *i.e.* along the X axis; these abscissae need not be actually 1, 2, 3, 4, 5 cm. long, but they must be *proportional* to these lengths, *i.e.* if the radius, 3 cm., is represented by an abscissa 15 mm. long, the radius 4 cm. must be represented by an abscissa 20 mm. long. You are free to choose your own scale. As a rule it is best to choose such a scale that the curve is as large as possible for the piece of squared paper provided, since your percentage error will then be as small as possible. The scale chosen for the ordinates need not be the same as that selected for the abscissae. An area of 1 sq. cm. can be represented by a length of 1 mm., 5 mm., 20 mm., or any other value, according to the size of the piece of squared paper and the magnitude of the greatest abscissa.

9. Plot the curve whose equation is xy = constant. To do this, take a number (*e.g.* 60) that can readily be resolved into several pairs of factors. Suppose xy equal to this number. If $xy = 60$, we have the following pairs of factors :—

When										
$x =$	1	2	3	4	6	10	15	20	30	60
$y =$	60	30	20	15	10	6	4	3	2	1

Plot these co-ordinates and join the points by a curve. This curve is called a *rectangular hyperbola*.

10. Plot a curve between the pressures and volumes observed in the Boyle's Law experiment. Since the results show that $p_1v_1 = p_2v_2 = p_3v_3 \ldots$, very nearly it follows from the above example that the curve obtained is a *rectangular hyperbola*.

CHAPTER IX.

THE SIMPLE LEVER.

79. DEFINITION.—A *lever* is a rigid rod or bar capable of turning freely about a fixed point of support. It is used to enable a force applied at one point to overcome a resistance acting at another point.

Examples.—A see-saw, the common balance, a crow-bar used for lifting a weight, an oar used in rowing a boat, the human arm supporting a load in the hand.

The force applied is called the **effort.** The force overcome is called the **resistance.** The point about which the lever turns is called its **fulcrum,** and the ratio between the resistance and the effort is called the **mechanical advantage** of the lever. In its simplest form the lever is a straight rod, the two arms therefore being in one straight line, which is usually horizontal. The resistances are nearly always weights.

80. The Principle of the Lever.—This is best deduced from the results of experiments.

Exp. 35.—*Make a simple lever.*—A simple form of lever can be made from a half-metre scale graduated in millimetres. To prepare it, bore clean circular holes about a millimetre or two millimetres in diameter at every 5 cm. division along the scale (Fig. 57). It is convenient to bore them not along the central line but nearer the upper edge. To make a fulcrum, drive a stout sewing needle or a thin smooth wire nail into the edge of the bench, or a board fixed in the wall, or a piece of wood which can be held in a strong clamp. Other apparatus required for the experiment includes some weights, say ounce weights and fractions of a pound, making up to two pounds, provided either with rings or suitable projections for the purpose of suspension by strings, a spring balance reading in ounces up to two pounds or more, and some cotton or fine string.

You may consider that a spring balance is graduated by hanging on different weights and marking the position of the pointer on the face of the instrument for each weight by the number denoting the number of pounds in that weight. For instance, when an 8-ounce weight is put on, 8 ozs. is marked against the position of the pointer. Obviously, if now we pull the spring out till the pointer marks 8 ozs. we are exerting a force equal to the weight of 8 ozs. on the spring, or, we may say, the spring is pulling at our hand with a force equal to the weight of 8 ozs.

If ounce or fraction of a pound weights are not available lumps of iron or lead, iron tools, or anything else of convenient size may be taken and weighed by the spring balance. They can then be used as weights.

Instead of ounce and pound weights and a spring balance reading up to two pounds or more, a set of gramme weights beginning with 50 grammes and going up to a kilogramme and a balance reading similarly may be used.

Exp. 36.—*To deduce the principle of the lever when the weights are hung on either side of the fulcrum.*

Balance the lever (made as above described) with its central hole C (Fig. 57) supported by the needle. If it does not balance level tie a piece of fine wire tightly round the lighter end adjusting the length or position of the wire until it does balance level.

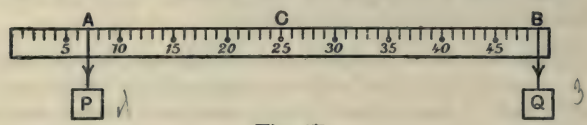


Fig. 57.

Take two weights, P and Q (say $P = 4$ ounces, $Q = 3$ ounces), and suspend them by loops of cotton from points A and B on the lever; one on each side of C, so that P and Q tend to turn the lever in opposite directions. Place P, say, 18 cms. from C, then find exactly where Q must be placed to balance, *i.e.* to make the lever set horizontal; you will find that Q must be placed 24 cms. from C.

Observe that $4 \times 18 = 3 \times 24$,

i.e. $P \times AC = Q \times BC$.

Repeat the experiment with P and Q at different distances from C. The following distances may be tried for AC:—

$AC = 9$ cms., $13\frac{1}{2}$ cms., 18 cms., 27 cms.,

and you will find that

$BC = 12$ cms., 18 cms., 24 cms., 36 cms. respectively.

Now repeat the experiment with different weights, say $P = 8$ ozs., and $Q = 1$ lb.,

$P = 6$ ozs., $Q = 9$ ozs., etc.

In each case you will find that when P and Q balance

$$P \times AC = Q \times BC.$$

We may call AC the arm of P, and BC the arm of Q; our relation then becomes:—

The force on one side of the fulcrum multiplied by its arm is equal to the force on the other side of the fulcrum multiplied by its arm.

This is the *Principle of the Lever.*

N.B.—We have here neglected to take account of the weight of the scale, because the two sides balance each other.

The product of the force P into its arm AC is called the **turning moment** of P about C, or simply the **moment** of P about C. It follows from this that when two forces balance on a simple lever, as in Fig. 57, the moments of the two forces about the fulcrum are equal.

Example.—A piece of porcelain weighing 1 lb. has been accidentally broken in two pieces. How would you find the weight of each piece without using an ordinary balance?

Set up a simple lever, as in the last experiment. Call the fragments P and Q. Suspend P and Q by cotton loops from the lever and move the loops up and down until P and Q balance. Measure the distances AC, BC, denote them by x and y.

Then since $P \times x = Q \times y$ $\qquad Q = \frac{x}{y}P,$

but $\qquad P + Q = 1$ lb. $\qquad \therefore P\left\{1 + \frac{x}{y}\right\} = 1$ lb.

$$\therefore P = \frac{y}{x+y} \text{ lb.} \quad \text{and} \quad Q = \frac{x}{x+y} \text{ lb.}$$

For example, $\qquad$ if $x = 4$ ins. and $y = 5$ ins.

then $\qquad P = \frac{5}{9}$ lb. and $Q = \frac{4}{9}$ lb.

Exp. 37.—*Repeat* **Exp. 36,** *using more than one weight on each side of the fulcrum.*—Set up a simple lever as before. The same lever may be used, but as it would be rather crowded with more than one weight on each side, it would be rather better to use a metre scale with a hole bored through at the 50 cm. mark.

You will find that, when the lever balances level, the sum of the moments of the forces on one side of the fulcrum is equal to the sum of the moments of the forces on the other side of the fulcrum.

Set down the results in the following manner:—

Left side of fulcrum.			Right side of fulcrum.		
Weights.	Distances from fulcrum.	Moments.	Weights.	Distances from fulcrum.	Moments.
25 gms.	22 cms.	550 units	35 gms.	24 cms.	840 units
60 „	9 „	540 „	50 „	5 „	250 „
		1090 „			1090 „

When the lever is uniform the weight of the portion on one side of the middle point is exactly equal to the weight of the portion on the other side, so the whole weight may be considered to act vertically downwards through the middle point. If the fulcrum is at the centre the weight of the lever does not tend to turn it in either direction. But if the fulcrum is not at the centre this weight has a turning effect which must be taken into consideration.

Suppose the lever weighs 92 grammes and the fulcrum is 10 cms. from the middle point, the lever tends to turn round the fulcrum, the moment of its weight being 920 units, and to preserve equilibrium a force with an equal moment must be applied to the other side.

Exp. 38.—*Without using a balance find the weight of a uniform lever.*—Take the half-metre scale of **Exp. 36**, hang it with the fulcrum F at the hole on the 5 cm. mark (Fig. 58). Hang a weight P on the short arm, and shift it along the lever until the lever

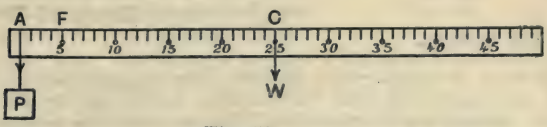


Fig. 58.

balances horizontally under the combined action of P and the weight of the lever. Suppose P then hangs at A. The weight W of the scale acts downwards at its middle point C. Hence, applying the Principle of the Lever, we get

$$P \times AF = W \times FC.$$

$$\therefore\ W = P \times \frac{AF}{FC}.$$

In the case under consideration $FC = 20$ cms. Suppose that $P = 10$ ozs. and $AF = 4$ cms., then

$$10 \times 4 = W \times 20.$$

$$\therefore\ W = 2 \text{ ozs.}$$

Repeat the experiment, using the hole at the 10 cm. mark for the fulcrum. You may find that if $P = 8$ ozs., $AF = 3{\cdot}7$ cms., whence

$$8 \times 3{\cdot}7 = W \times 15,$$

or

$$W = \frac{29{\cdot}6}{15} = 2 \text{ ozs. approx.}$$

Repeat the experiment, using in turn the 15 cm. hole, the 20 cm. hole, etc., for the fulcrum.

Verify the result by direct weighing on an ordinary balance or a spring balance.

81. Systems of Levers.—Levers are divided into three systems according to the relative positions of the fulcrum and the points of application of the effort and the resistance.

N.B.—In Figs. 59-64 the weight of the lever is neglected; in most practical applications of the lever its weight is small compared to the other forces.

1st System.—The fulcrum is between the points of application of the effort and the resistance. In Fig. 59 E and R are the effort and resistance and the bar rests on a wedge as a fulcrum at F. The simple lever of **Exps. 35-37** belongs to this system.

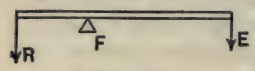


Fig. 59.

Common Examples.—The see-saw. The common balance (Fig. 18). The steel-yard (Fig. 20). The crowbar used as in Fig. 60: here the effort is the force P exerted at A by the man; the resistance is the force Q exerted by the block of stone on the end of the bar; the fulcrum is the point C, where the bar rests on the log.

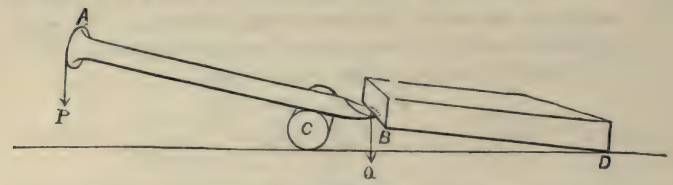


Fig. 60.

2nd System.—The point of application of the resistance is between the fulcrum and the point of application of the effort. In Fig. 61, E is the effort, R the resistance, and F the fulcrum.

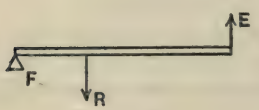


Fig. 61.

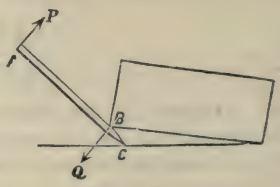


Fig. 62.

Common Examples.—The crowbar when used as in Fig. 62. A punching machine. An oar used in rowing; here the effort is applied by the rower at the handle; the resistance is that of the boat, and acts on the oar at the rowlock. The fulcrum is the blade, which is practically stationary while in the water. The student should, if possible, confirm this last statement for himself.

3rd System.—The point of application of the effort is between the fulcrum and the point of application of the resistance. In Fig. 63, E is the effort, R the resistance, and F the fulcrum. In this case the bar is not resting on the fulcrum at F, but fastened down to it in some way; for otherwise this end of the bar will be lifted off F by the force E.

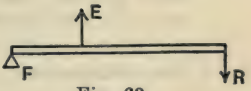


Fig. 63.

Common Examples.—The treadle of a turning lathe (Fig. 64). The human arm supporting a weight in the hand. Here the effort is exerted by means of a tendon attached to the fore-arm a little below the elbow; the elbow is the fulcrum, and the weight placed in the hand is the resistance. Draw a diagram for yourself.

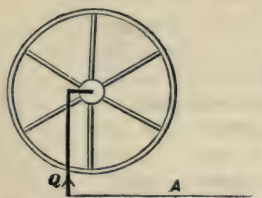


Fig. 64.

Exp. 39.—*Make a simple lever of the second system.*—Take either the half-metre scale of **Exps. 35, 36,** or the metre scale of **Exp. 37.** We may adopt one of two plans:—

(1) Balance the scale on its middle point, in which case we do not take account of the weight of the scale.

(2) Use for a fulcrum a hole near the end of the scale, in which case we have to allow for the weight of the scale.

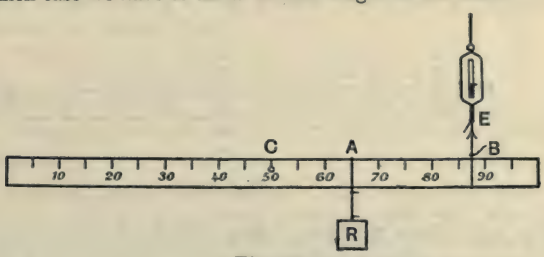


Fig. 65.

Exp. 40.—*Using the lever of* **Exp. 39** (1), *find the law of a lever of the second system.*—Take the metre scale and hang it with its middle point C on the fulcrum (Fig. 65). Slip two loops of

cotton over one end. Hang a convenient weight R, say, on the inner loop and place the loop at a point A, say, on the lever. To the other loop attach a spring balance, pulling upwards, and lift the balance until the balance and string are vertical and the lever horizontal. If the balance does not record an exact number of ounces, shift it along the lever until it does. Read the balance and its distance from C, viz. BC. Let E be the force exerted by the balance. Show by your observations that the moments about C are equal, *i.e.*

$$R \times CA = E \times CB.$$

For example, in one experiment,

$$CA = 15 \text{ cms.},$$
$$BC = 37\tfrac{1}{2} \text{ cms.},$$
$$R = 16 \text{ ounces},$$
$$E = 6\tfrac{1}{2} \text{ ounces (very nearly)},$$
$$R \times CA = 16 \times 15 = 240,$$
$$E \times BC = 6\tfrac{1}{2} \times 37\tfrac{1}{2} = 244.$$
$$\therefore R \times CA = E \times CB \text{ (approximately).}$$

Repeat the experiment, using different values of R, of CA, and of BC, and show that all your results yield to the same conclusion, viz. that the upward moment of the effort around C is equal to the downward moment of the resistance R about the same point.

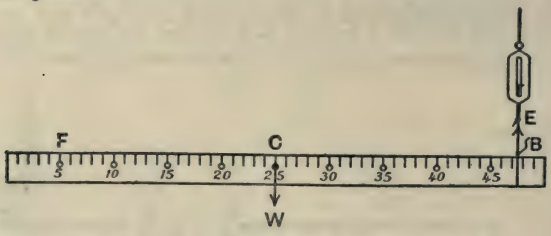


Fig. 66.

Exp. 41.—*Using the lever of* **Exp. 39** (2), *find its weight without using an ordinary balance.*—Place the fulcrum F at the 5 cm. hole (Fig. 66) and support the long end by a spring balance which is capable of reading accurately to a much smaller weight than that of the lever. When the lever is horizontal, take the reading of the balance, E.

Then, by the principle proved above, we get

$$E \times BF = W \times FC = W \times 20,$$

$$\therefore W = \frac{E \times BF}{20}.$$

Thus, for example, in a certain experiment (Fig. 66), E was 1 oz. and BF 42·5 cms., then

$$W = \frac{1 \times 42{\cdot}5}{20} = 2{\cdot}1 \text{ ozs.}$$

Exp. 42.—*Using the lever of* **Exp. 39** (2), *verify the principle of the lever when there are three forces tending to turn the lever about its fulcrum.*—Knowing the weight of the lever, we can use the spring balance to balance the weight of the lever and any other weight placed at any other point.

To do this repeat the last experiment, hanging on an additional load P at any point A. Adjust the spring balance till the lever is horizontal and read all the distances and the spring balance.

Then the upward turning moment

$$= E \times BF \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1),$$

and total downward turning moment

$$= W \times CF + P \times AF \quad \ldots\ldots\ldots\ldots\ldots\ldots(2).$$

Show from the figures which you obtain by experiment that (1) and (2) are equal.

In an experiment with a half-metre scale weighing 2 ozs., fulcrum at F, P was a load of 8 ozs. placed 6 cms. from F, and it was found that the spring balance read 2 ozs. when its loop of cotton was 44 cms. from F.

Then upward moment $= 2 \times 44 = 88,$

downward moment $= 2 \times 20 + 8 \times 6$

$= 88.$

These two moments are equal, which verifies the Principle of the Lever.

Exp. 43.—*To verify the principle of the lever with a lever of the third class.*—Modify either of **Exps. 39, 40, 41,** placing the spring balance closer to the fulcrum than the weights it supports. For example, in an experiment with a half-metre scale balanced at its centre it required a force of $2\frac{1}{2}$ lbs.-wt. exerted by a spring balance at a distance of 3 cms. from the centre to support a $\frac{1}{2}$ lb.-wt. hanging 15 cms. from the fulcrum. Show that this verifies the principle of the lever.

82. The **Principle of the Lever**, which we have proved in the foregoing experiments, may be expressed as follows. The principle should be learnt by heart.

When there is equilibrium in the lever, the effort multiplied by its perpendicular* distance from the fulcrum is equal to the resistance multiplied by its perpendicular distance from the fulcrum.

When there is a slow motion, the forces may be regarded as nearly or quite in equilibrium, and the above equation still holds.

When the weight of the lever cannot be disregarded, we have *three* forces to consider, instead of two; and therefore we have *three moments round the fulcrum*, which must balance for equilibrium. Similarly other extra forces acting on the lever increase the number of moments to be taken into consideration.

83. Mechanical Advantage of Straight Levers.—Referring to Figs. 57-66, we see that in each kind of straight lever, on taking moments round the fulcrum, we get

$$\text{effort} \times \text{its arm} = \text{resistance} \times \text{its arm}.$$

$$\therefore \textbf{mech. adv.} \text{ which is equal to } \frac{\textbf{resistance}}{\textbf{effort}} \text{ (§ 79)}$$

$$\text{is equal to } \frac{\textbf{arm of effort}}{\textbf{arm of resistance}}.$$

In Class I. (Fig. 59) the arm of the effort can be greater or less than the arm of the resistance;

$\therefore$ the mechanical advantage can be greater or less than unity.

---

*The moment of a force about the fulcrum is the product of the force and the *perpendicular* drawn from the fulcrum to the line of action of the force. In all the preceding experiments this perpendicular is equal to the actual length measured along the lever, but if the lever were not horizontal or the forces not vertical, the lengths measured along the lever could not be used: the actual perpendiculars would then have to be measured.

There is one apparent exception to this statement. If the lever is straight and the forces are all parallel, but not perpendicular to the lever, then the lengths measured along the lever are *proportional* to the actual perpendiculars and may be used instead of them.

By placing the fulcrum very close to the resistance the mechanical advantage becomes very great, so that a very heavy body could then be raised with a small effort. This is what Archimedes meant when he said: "Show me where I may rest my lever and I will move the world."

In Class II. (Fig. 61) the arm of the effort is always greater than the arm of the resistance;

$\therefore$ the mechanical advantage is always greater than unity.

In Class III. (Fig. 63) the arm of the effort is always less than the arm of the resistance;

$\therefore$ the mechanical advantage is always less than unity.

In machinery this class is only used when a slow motion is to be converted into a rapid one. It occurs for this purpose in many of the limbs of animals. For example, in the human forearm, if the tendon is shortened by $\frac{1}{2}$ in., the hand may be raised through as much as 2 ft.; thus a slow motion of the contracting muscle gives a rapid motion to the hand.

84. The Common Balance.*—The common balance is a simple lever with the fulcrum at its middle point. To avoid the need of calculation the masses are so placed that the lines of action of their weights are equidistant from the fulcrum. If l be the length of each arm, and an unknown mass of x grammes in one pan is balanced by masses amounting to 12 grammes in the other pan, then

$$x \times l = 12 \times l$$

$$\therefore x = 12 \text{ grammes.}$$

In accurate balances one arm is divided into ten equal parts, and a movable mass of 10 milligrammes, called a *rider*, can be placed at any point along it. Suppose the rider is at the third division from the fulcrum and let l denote the length of the arm. Its turning moment in that position is $\frac{3}{10} \times l \times 10 = 3l$, *i.e.* it has the same effect there as a mass of 3 milligrammes would have in the pan. The rider makes it unnecessary to use masses smaller than one centigramme in the pan.

* Details of the construction of a common balance are given in Ch. IV.

Summary.—Chapter IX.

1. A *lever* is a simple contrivance by which force applied at one point is made to overcome a resistance at some other point. (§ 79.)

2. The *mechanical advantage* of a lever is the ratio of the resistance to the applied force (or effort). (79.)

3. There are three kinds of levers—

In the *first* kind the fulcrum is between the effort and the resistance.

In the *second* kind the resistance is between the effort and the fulcrum.

In the *third* kind the effort is between the resistance and the fulcrum. (§ 81.)

4. The *principle of the lever* is

Effort × its arm = Resistance × its arm. (§ 82.)

5. The common balance is a simple lever of the first system with its fulcrum midway between the points of suspension of the two scale pans. (§ 84.)

EXERCISES XII.

1. What do you understand by the terms *moment*, *fulcrum*, *lever*?

2. Without using a balance find the weight of a uniform lever. Use this lever to find the weight of a glass stopper. Verify by a direct weighing.

3. A metre scale is pivoted at its middle point and is in equilibrium in a horizontal position. It remains balanced horizontally when pairs of weights are hung from it to the left and right of the pivot at the marks stated below—

Left, 50 gms. at 0 cms. 80 gms. at 40 cms. 75 gms. at 40 cms.
Right, 100 gms. at 75 cms. 40 gms. at 70 cms. 25 gms. at 80 cms.

What conclusion do you draw from these three results?

4. If you were provided with some thin sheet lead, a 10 gramme weight, and a balance known to have unequal arms, how could you make from the lead another 10 gramme weight?

5. With the given rod and a piece of lead make and graduate a common steel-yard. (See § 29 for a description of the steel-yard.)

6. Show that, if two forces acting on a simple lever are in equilibrium, they are inversely proportional to their distances from the fulcrum. Forces of (i.) 2 lbs. and 3 lbs., (ii.) 5 lbs. and 7 lbs., balance each other at the ends of a lever of the first class, whose length is $2\frac{1}{2}$ ft. Where is the fulcrum in each case?

7. A weightless lever AB, of the first order, 8 ft. long, has the fulcrum 2 ft. from B; a weight of 5 lbs. is hung from A and one of 17 lbs. from B. From what point must a weight of 2·5 lbs. be hung to keep the lever horizontal?

8. Let AB be a horizontal line 10 ft. long, and F a point in it 6 ins. from A; suppose that AB is a lever that turns on a fulcrum under F, and carries a weight of 50 lbs. at B. If it is kept horizontal by a fixed peg above the rod, 5 ins. from F and 1 in. from A, find the force on the fixed peg.

9. A pair of nutcrackers is 5 ins. long, and when a nut is placed $\frac{7}{8}$ in. from the hinge a force of $3\frac{1}{2}$ lbs. applied at the end will crack the nut. What weight, if simply placed on the top of the nut, would crack it?

10. A nut which is capable of resisting a direct force of 80 lbs. is placed in a pair of nutcrackers, $1\frac{1}{2}$ ins. from the fulcrum. A force of 16 lbs. is exerted at the end, and just cracks the nut. What is the length of the nutcrackers?

11. A man raises a cube of granite, whose side is 4 ft., which weighs 4 tons, by means of a crowbar 6 ft. long, by thrusting one end of the crowbar a distance of 4 ins. under the stone. What force does he exert at the other end? What is the mechanical advantage?

12. A weight of 35 lbs. balances a weight of 15 lbs. at the extremities of a uniform lever 15 ft. long. Find the lengths of the arms.

13. If one end of a bar rests on a beam, and a weight of 60 lbs. be suspended from it one-fifth of its length from the beam, what effort at the other end will support the weight?

14. A man who weighs 160 lbs., wishing to raise a rock, leans with his whole weight on one end of a horizontal crowbar 5 ft. long, propped at a distance of 4 ins. from the end in contact with the rock. What force does he exert on the rock, and what is the mechanical advantage?

15. Explain, with diagrams, the kind of lever employed in each of the following actions—(*a*) opening an oyster with an oyster-knife, (*b*) opening a sardine tin with a tin-opener, (*c*) punching a tram-ticket, *d*) shutting a door by pushing close to the hinges.

16. What is meant by the mechanical advantage of a lever? Describe any simple device by means of which you could just support a weight of 5 pounds, using only a one-pound weight.

CHAPTER X.

FORCE. WEIGHT. GRAVITY.

85. Mechanics defined.—Branches of Mechanics.—The name **Mechanics** was originally used to designate the science of making machines. It is now, however, very generally applied to the whole theory which deals with motion and with bodies acted on by forces.

The subject Mechanics is generally divided into two parts—

(1) **Statics,** which treats of bodies kept at rest under the action of forces;

(2) **Dynamics,** which treats of moving bodies.

86. Particle.—DEFINITION.—*A* **particle** *is a portion of matter whose volume is so small that we can altogether disregard its form, and consider it merely as a portion of matter collected at a single point.*

87. Rest and Motion.—DEFINITION.—A particle that during a certain interval occupies the same position in space is said to be **at rest** during that interval; if at one instant it occupies a certain position, and at another instant a different position, it is said to be **in motion.** Motion is **therefore** equivalent to **change of position.**

88. Force.—DEFINITION.—Force is that which changes, or tends to change, a body's state of rest or motion.

The words *tend to change* are necessary; for the force may not actually cause any change, as its effect may be neutralised by one or more other forces acting on the body at the same time.

The above is the usual definition of force; but the idea of force is really fundamental, and cannot be satisfactorily defined.

When we push against a body or pull at it we exert force on it. If we support a lump of iron on the outstretched hand, it requires muscular effort to prevent the iron from dropping to the ground, because the Earth attracts the iron with a force which we call its weight. It is evident that the force which the hand exerts upwards must be equal to the force with which the Earth pulls the iron downwards.

If, instead of supporting the lump of iron by the hand, we place it on a table, the table exerts an upward force equal to that which the hand had previously exerted. In this case the force is supplied by the natural resistance of the wood of the table to being broken or distorted.

The iron may also be suspended by a string. In this case the upward force is that due to the natural resistance of the string to being elongated. The string is said to be in *tension*.

If, instead of a string, a coiled spring, such as that in a spring balance, is used to support the iron, it will be observed that the force of the Earth on the iron elongates the spring to a definite extent. In this case the force exerted by the Earth on the iron is greater than the force which the unstretched spring is able to exert, and equality is not established until the spring has been elongated to a certain extent.

In each of the above cases the lump of iron is said to be **in equilibrium** under the action of two forces—the downward force due to the earth's attraction, and the equal upward force due to the hand, the table, the string, and the spring respectively. In Statics equal forces are defined as those which, if acting in opposite directions upon the same particle, would keep it in equilibrium.

89. Dynamical Treatment of Force.—When a body originally at rest is acted upon by a force whose effect is not neutralised or modified by other forces the body begins to move, and, if the force continues to act, the motion becomes faster and faster.

If the same force acts successively on bodies of different mass, it will generate more velocity in a smaller mass than it will in the same time in a larger mass. We may define two

forces as *equal* if when they are applied to bodies of the same mass for equal intervals of time they import the same motion or change of motion to these bodies.

That the dynamical definition of force is equivalent to the statical can be seen from the following considerations:—If two equal forces act on the same particle in exactly opposite directions, the motion which one force tends to impart is exactly the reverse of that which the other tends to impart. The particle cannot move in opposite directions at the same time, and there is no reason why it should move in one direction rather than the other. Hence it will remain at rest, and the two forces will be said to *balance*, or be *in equilibrium*.

The dynamical qualities of force cannot be considered until the properties of motion have been investigated. This is beyond the scheme of the present book.

90. Mass and Weight.—The terms **mass** and **weight** have been explained in Chapter IV. We have there defined the *mass* of a body as the *quantity of matter* in the body and the *weight* of the body on the surface of the Earth as the *force with which the body is attracted to the Earth.* It was also stated that weight is proportional to mass.

If we keep to one kind of matter—say iron—we may say that both mass and weight are proportional to volume: *e.g.* 2 cubic inches of iron have twice the mass of, and will weigh twice as heavy as, 1 cubic inch of iron.

We may thus compare the weights of different lumps of iron by simply measuring their volumes.

If, however, we wish to compare the weight of a lump of iron with the weight of a lump of coal, we are met by a difficulty. The kind of matter in each is different. But the difficulty may be solved by the use of the Ordinary Balance and the Spring Balance.

91. The Spring Balance or Dynamometer.—The ordinary balance and the spring balance have been described in Chapter IV. The principle of the ordinary balance was also dealt with in Chapter IX.; that of the spring balance will be understood better after performing Exps. 46-49 of Chapter XI.

A spring balance is often called a dynamometer. The term **dynamometer** simply means a force measurer. An ordinary balance measures only a particular kind of force, viz. weights. Since, however, spring balances can be used in any position, they can be used to measure forces in general, *e.g.* if I hold a spring balance in my left hand and extend the spring with the other till the pointer reads one pound, my right hand is exerting on my left hand, and therefore also my left hand on my right hand, a force equal to the weight of one pound.

92. Gravity.—It is a matter of observation that most bodies when removed from contact with other bodies fall to the Earth. This movement is due to **gravity**, *i.e.* to the attractive force exerted by the Earth on the bodies.

There are, however, some bodies which, when free to move, *ascend* instead of descend. A balloon filled with coal-gas is a well-known example of this. This upward motion is due to the fact that the force of gravity downwards is counteracted by another and a greater force *upwards*, due to the presence of the air, just as when a cork is placed in water the upward force due to the water displaced is greater than the weight of the cork.

If we take a lump of lead, a piece of paper, and a feather and release them from the hand at a height, say, of 5 ft. from the floor, they all fall to the ground in consequence of the force of gravity acting upon them. The lead reaches the ground very soon, the paper not quite so soon, while the motion of the feather is, in comparison, very slow.

Aristotle taught that bodies fall at rates depending on their weights: that the heavier a body is the faster it should fall. Galileo disproved this (in 1590) by a neat argument. He said: "If, therefore, two bodies weighing one and nine pounds respectively are tied together, the lighter one, which falls more slowly, must retard the heavier one. But the two together weigh ten pounds, and should therefore, *ex hypothesi*, fall even faster than that weighing nine pounds; so that Aristotle's view leads to an absurdity—avoidable only by the assumption, proved by experiment from the Leaning Tower of Pisa, that all bodies fall with equal velocity unless they are so light as to be impeded by the air resistance."

93. Resistance of Air.—That the difference in the rate of falling of bodies of very different density and structure is due to the presence of the air was shown conclusively by an experiment first performed by Newton, and now called the **Guinea and Feather experiment.** Newton took a long glass tube (Fig. 67), about 3 inches in diameter, closed at one end. A guinea and a feather were then inserted, and the other end closed with an air-tight cap and a stop-cock. When the tube was inverted it was found that the times of falling were very unequal. The stop-cock was next attached to an air-pump and the air exhausted. The tube was then detached from the air-pump and the experiment repeated, and it was found that the guinea and feather moved side by side down the tube with equal velocities.

Fig. 67.

The same thing can be shown more simply without an air-pump, by the following experiments, which should be performed by the student before proceeding further:—

Exp. 44.—Take two equal masses of tin, about 1 oz. each, one in the form of a spherical ball and the other in the form of a very thin circular plate. Then drop them from the same height, holding the plate horizontally, and it will be found that the plate takes a longer time than the ball to reach the ground. This difference cannot be due either to the *material* or to the *mass*, for they are the same for both bodies, but clearly arises from the fact that the plate has a larger amount of air to move out of the way. If now the plate be held *vertically*, so that it exposes only a small amount of surface to the air in the direction of its motion, and the experiment is repeated, no difference in the times of falling can be detected.

Exp. 45.—Take a small tin canister without the lid (*e.g.* a cocoa-tin), and in it place various objects, such as a coin, a feather, a piece of thin tissue paper, etc. Drop the canister from a height. All the objects will remain inside and will reach the ground together, showing that all are equally acted on by gravity.

Deductions.—(1) The resistance of the air causes bodies dropped simultaneously from the same height to reach the ground at different instants; (2) if the resistance of the air be removed, the bodies will reach the ground simultaneously.

94. Weight Proportional to Mass.—It is now clear that *in vacuo* (*i.e.* in a space from which the air has been removed) all bodies move towards the Earth with velocities which (i) are equal at the ends of equal intervals of time from rest, and (ii) which increase at the same rate. In other words:

The acceleration due to gravity is the same for all bodies *in vacuo* at the same place on the Earth's surface.

The reason that the acceleration due to gravity is the same for all bodies is that the weight of a body (*i.e.* the force with which the Earth attracts the body) is proportional to its mass. The weight of a mass of two grammes is twice the weight of a mass of one gramme; that is, the force causing a two-gramme mass to fall is twice that causing a one-gramme mass to fall; but the mass on which the force acts is also twice as great, so that the acceleration is the same in the two cases.

Reversing this argument, the experimental verification of the fact that the acceleration due to gravity is the same for all bodies proves that weight is proportional to mass. Galileo performed his experiments by dropping simultaneously *two different weights* from the top of the Leaning Tower of Pisa. They reached the ground simultaneously. He interpreted this correctly, and was thus the first to enunciate that weight is proportional to mass.

Galileo's statement may also be verified by showing that the *time of swing of a pendulum bob* is the same whether the bob is hollow or filled with material of any density. This method was originally used by Newton.

Galileo further illustrated his point by *rolling spheres down inclined planes*, showing that the acceleration, though dependent on the dimensions, etc., of the rolling bodies and the slope of the planes, was independent of the masses.

95. Weight Varies with Locality.—The experiments quoted above show that at the same place weight (*i.e.* the force of attraction of the earth for a body) is proportional to mass. Now the shape of the Earth is not exactly spherical. It is flattened at the poles, so that a body at the poles is nearer the centre of the earth than a body at the equator. On this account the weight of a body is **greater at the poles than at the equator.** Similarly the weight of a body decreases as we ascend a high mountain.

Again, the Earth is rotating on its axis, the velocity due to rotation being greatest at the equator. The so-called "centrifugal force" due to this rotation tends to diminish the weight of a body; hence on this account also the weight of a body at the equator is less than its weight at the poles. **The weight of a body therefore depends on the locality at which it is placed. The mass of a body is of course the same all the world over.**

96. Weighing by a pair of scales and a spring balance.—The variations referred to above in the weight of a body cannot be detected, however, by weighing it with a pair of ordinary scales; for the "weights" that are used are equally affected by the variation of the Earth's attractive force. Whatever change takes place in the force with which the body weighed presses upon the pan, the same change will appear in the force with which the "weights" press upon their pan, and consequently the body and the weights will still balance. Hence **masses, not weights, are compared by a pair of scales.**

With this method of weighing it is important to notice that, if we weigh a pound of sugar at the Equator and another at the North Pole, we should get the same quantity of sugar in each case, *although the attractive force of the Earth is less at the Equator than at the Pole.*

For, if a certain quantity of sugar balances the leaden weight at the Equator, when taken to the Pole both the sugar and the lead will weigh more, and the same quantity of sugar will still balance the same quantity of lead.

With a spring balance, however, the case is different. If a certain force overcomes the elasticity of the spring to such an extent as to depress the pointer 1 in., that force will depress it 1 in. at any other place on the Earth; for the change in the attractive power of the Earth does not affect the elasticity of the spring. When, therefore, we find, as we should, that a body attached to the spring depresses the pointer to different distances at two different places, we infer that the weight of the body has changed. From this we see that a **spring balance compares weights, not masses.**

In this case, if we weigh a pound of sugar at the Equator, we should get more than if we weighed it at the Pole. For, since the sugar weighs less at the Equator, more of it will be required to stretch the spring to a given length than would be required at the Pole.

NOTE.—In practice the variation of the Earth's force of attraction is so small that only the most sensitive spring balance will detect it.

97. Statical Units of force.—The weight of a body is a force, and accordingly the most convenient *units of force* are weights.

In the **English** system the **unit of force** is the weight of a pound and is called a force of **1 pound-weight.** Larger forces may, however, be measured in hundred-weights or ton-weights.

If the **C.G.S.** system is used, the statical unit of force will be the **weight of a gramme** or of a kilogramme (1,000 grammes), according to which is the most convenient.

Summary.—Chapter X.

1. Force is that which changes, or tends to change, a body's state of rest or motion. (§ 88.)

2. Equal forces are defined—in Statics—by saying that, if they act in opposite directions upon a particle, the particle remains in equilibrium. (§ 88.)

3. Equal forces are defined—in Dynamics—by saying that, if they act for the same time upon equal masses originally at rest, they will produce in each the same motion. (§ 89.)

4. A spring balance or dynamometer measures force. It is usually employed to measure the particular kind of force known as weight. (§ 91.)

5. At the same place all bodies fall with the same acceleration—if the resistance of the air be removed. From this it follows that at the same place weight is proportional to mass. (§§ 93, 94.)

6. The time of swing of a pendulum with a hollowed bob, first empty and afterwards filled with different materials, is always the same at the same place, again proving that weight is proportional to mass. (§ 94.)

7. The *mass* of a body is the same all the world over, but the *weight* of a body is subject to small variations as the body is moved about to various places. (§§ 95, 96.)

8. The most convenient units of force are weights: *e.g.* the pound-weight, the gramme-weight, etc. (§ 97.)

EXERCISES XIII.

1. What is *force*? How do you define equal forces (1) statically, (2) dynamically?

2. Define *mass* and *weight*. Why is the mass of a body the same all the world over, while the weight of a body changes?

3. What does an ordinary balance compare? What does a spring balance measure?

4. Describe an experiment to show that *in vacuo* all bodies fall at the same rate.

5. Why can a spring balance be used more generally as a measurer of force than an ordinary balance?

6. What are the statical units of force?

CHAPTER XI.

MOLECULAR PHYSICS: ELASTICITY: CAPILLARITY.

Cohesion and Adhesion.

98. Molecular Motions.—The ultimate particles of any body—the *molecules*, as they are called—are in a state of rapid motion. In the case of a **gas**—in which case our knowledge of what is happening is greatest—the particles are darting about hither and thither with an enormous velocity, colliding with one another and with the walls of the containing vessel. In fact this continual bombardment of the walls of the vessel by the particles constitutes the pressure of the gas.

When a gas is heated its particle velocity is increased. If the volume of the gas is unaltered so that there are the same number of particles in the same space, the increased velocity of the particles causes them to strike the walls of the vessel more frequently and with greater violence; so that **an increase of temperature must result in an increase of pressure.** If now the temperature of a gas is unaltered but its volume is increased, it follows that the particle velocity is unaltered while the number striking a given area of the walls (say a square centimetre) is decreased, for there are now fewer particles in the same space. Thus the force on this square centimetre is decreased, or, in other words, the pressure of the gas is decreased. Thus **an increase of volume results in a decrease of pressure.** Compare Boyle's Law, § 75.

The particles in **solids** and **liquids** are also in rapid motion, but their freedom is much more restricted than in the case of a gas. The particles are packed together much more tightly, so that collisions between particle and particle are very frequent and the particles do not have so much chance of changing their position. In the case of solids the motion is so restricted that the particle practically maintains the same position for all time, except for minute oscillations to one side or the other. In the case of liquids the motion is intermediate between that occurring in solids and gases, so that in time a particle will find itself in a very different position to that in which it was originally placed, but the motion is much slower than in the case of a gas. In solids and liquids, as in gases, a rise of temperature increases the particle velocity, and a fall in temperature diminishes it.

99. Molecular Attractions.—We know that the Earth attracts any body, and will cause it to fall if restraints are removed. The attraction is really mutual: the body moves towards the Earth, and the Earth also towards the body; but the motions are inversely proportional to the respective masses, so that practically speaking the Earth is at rest and it is only the particle that moves.

This, however, is only a special instance of a general law. Any two bodies attract one another with a force which depends upon their masses and the distance between them: in fact the force with which they attract one another is directly proportional to the product of the masses and inversely proportional to the square of the distance between their centres. Now in the case of the molecules of a body the masses are very small, so that unless they are very close to each other the attractive force is very small; but for particles which are very close together the attractive force becomes considerable.

We must therefore regard the equilibrium condition of the molecules of a body as due to a balance between the following two agents:—

(1) The motional energy of the particles which tends to separate them.

(2) The forces of attraction between particle and particle which tend to bring them together.

In solids the second influence is much the stronger, in gases the first.

If heat be imparted to a solid, the average speed of the particles is increased, and their tendency to separate is also increased, while separation diminishes the forces of attraction. If more heat is therefore supplied the tendency to separate increases further, and a time comes when the solid melts and a liquid results. If the heating is continued the motional energy increases, the forces of attraction decrease, and finally the liquid turns into a gas. It must of course be admitted that there are many solids which under ordinary conditions cannot be transformed into liquid and gas in this way: this is because at some stage of the heating chemical action takes place and the nature of the molecules themselves is then altered.

100. Cohesion, Adhesion, Chemical Affinity.—Cohesion is the name given to the system of attraction forces between the adjacent molecules of a body. It is cohesion which keeps the particles of a solid body from separating. Cohesion is greatest in solids and least in gases: in fact in gases it is practically zero. Cohesion is decreased by increase of temperature: it may also be altered in other ways, *e.g.* the coherence between particles of iron may be altered by *tempering*.

Adhesion is the name given to the system of attraction forces which act between the adjacent surface particles of two solid bodies placed in contact. In most cases the forces of adhesion are insignificant, but in some cases they are sufficient to hold the bodies firmly together. For Adhesion to be effective we must have intimate contact between the two surfaces. Thus, if two indiarubber surfaces are prepared, quite smooth, and then pressed together, union occurs and the two pieces of rubber form one piece. The same thing can be done with two clean lead surfaces. Adhesion is made use of in the operations of glueing, soldering and writing.

Chemical Affinity is the name given to the system of attraction forces which hold together the *atoms* of a molecule. (For the distinction between atoms and molecules see Section III., § 14.) It is conceivable that these forces

between the atoms may be of the same nature as the forces of cohesion and adhesion between the molecules, but such evidence as is at present available certainly does not justify us in making any such statement.

101. Tenacity of a Wire.—If a wire of any given material is used to support a load, and if this load is gradually increased, there comes a time when the wire breaks. The greatest load per square centimetre of cross section which the wire can support is called the **tenacity** of the material of the wire. The tenacity is thus a measure of the resistance which the material opposes to being torn asunder.

Substances vary greatly in their tenacities; thus a wire of lead of cross section 1 sq. cm. breaks under a load of 200 kilograms, while a similar wire of steel will bear 6000 kilograms. It is worth mentioning that the tenacity of steel is exceeded by the tenacity of fibres of unspun silk, for one of these will not break until the load reaches an amount equal to 50,000 kilograms per square centimetre of cross section.

Example.—Given that the tenacity of steel is 6000 kilograms per square centimetre, find the load which can be supported by a wire of diameter 1 millimetre.

The cross section of this wire is a circle of diameter 0·1 cm.

The area of cross section $= \pi \frac{d^2}{4} = \frac{3{\cdot}14 \times 0{\cdot}01}{4} = 0{\cdot}00785$ sq. cm.

When cross section is 1 sq. cm. greatest load $= 6000$ kilograms.

$\therefore$ When cross section is 0·00785 sq. cm. greatest load $= 600 \times 0{\cdot}00785$
$= 47$ kilograms.

Elasticity. Hooke's Law.

102. Elasticity.—Elasticity is a general name given to that property of a body in virtue of which it resists, and recovers from, change of shape or volume. All substances resist changes in volume and so have what is called *bulk elasticity*, but it is only solids that have elasticity of shape: no fluid—liquid or gas—can offer a permanent resistance to change of shape. On this property the definition of a fluid is based. (Ch. I., § 3.)

103. Extension of a Spiral Spring.

Exp. 46.—*To find how the extension of a spring depends upon the load which it supports.* Obtain a long spiral spring. Drive a nail into a stout wooden rod and support the rod vertically in a clamp and stand (Fig. 68). Hang a boxwood millimetre scale from the nail and then the spring in front of it. The lower end of the spring, after forming a loop, is twisted back and finally turned to form an index p pointing to the scale divisions ; or the end is passed axially through a cork that carries a needle (n) horizontally (Fig. 69). Suspend a scale-pan from the lower loop by a piece of string so that it hangs clear of the support, etc.

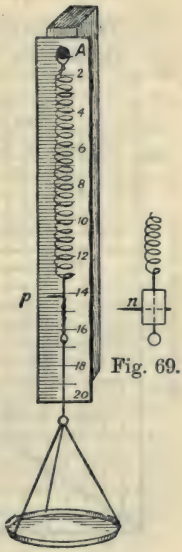


Fig. 69.

Fig. 68.

Observe the reading of the pointer, p, when the pan is unloaded, and when loaded successively with 20, 40, 60, etc., grammes, taking readings as the load is increased to a maximum, and also as it is decreased by the same steps. An error will occur in reading p if p is not almost in contact with the scale. If the spring hangs so that p is far from the scale, slightly rotate the spring on its vertical axis until p comes near the scale : then read.

The two sets of readings, *i.e.* that taken as the load is gradually increased, and that as the load is gradually decreased, should agree very well. If only slightly different the mean may be taken, if very different repeat the whole experiment.

Plot a curve taking loads as abscissae and extensions of the spring as ordinates.

The result will be somewhat as shown in Fig. 69, which represents a curve obtained in an actual experiment. The graph is practically a straight line through the origin of coordinates, and shows that the extension of the spring is proportional to the load applied. From the graph deduce the extension per

gramme and the weight required to produce unit, *i.e.* 1 cm., extension.

In the case represented in Fig. 70 the extension per 50 gms. $=3{\cdot}3$ cms., therefore the extension per gm. $=0{\cdot}066$ cm. The weight required to produce 1 cm. extension $=\frac{50}{3{\cdot}3}$ gms. $=15{\cdot}2$ gms. This could also have been read directly from the graph. It is the abscissa of the point A.

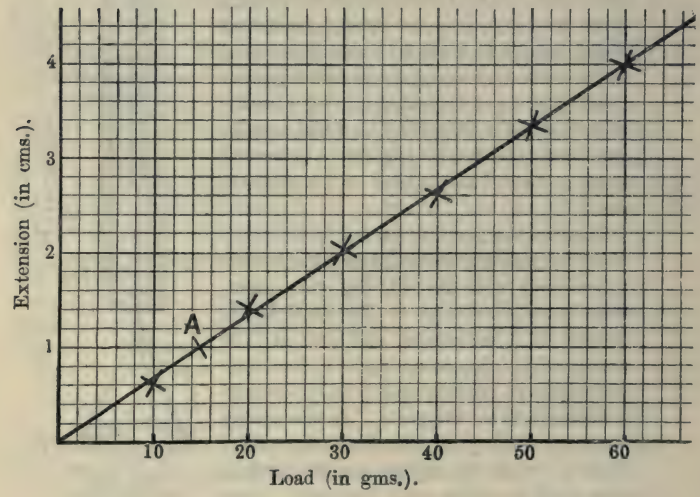


Fig. 70.

Exp. 47.—*Graduate a spring balance.* Remove the boxwood scale and fix, with drawing pins or small nails, a strip of paper behind the spring. Mark on the paper the position of the pointer for no load and for loads of 10, 20, 30, etc., grammes. Number the lines 0, 10, 20, etc., and subdivide the spaces between into halves or fifths.

We have thus made a scale to the balance, so that when the pointer points to any mark P we know that the load in the scale-pan is P grammes.

Exp. 48.—*Find by the spring balance of the last experiment the weights of coins, etc.* Place the body in the scale-pan; note the number of the division to which the pointer is drawn. Check by weighing on an ordinary balance.

Exp. 49.—*Find the specific gravity of glass, iron, coal, stone, sulphur, wax, methylated spirit, petroleum, salt solution, etc., by means of a spring.* (The experiment illustrates the use of Jolly's specific gravity balance.) Hang the spring as in Fig. 70. (Remove the scale-pan.) Observe the readings of the pointer, p, (i) when the spring is unloaded, (ii) when a body is hung from it in air, (iii) in water. The difference of the readings is the extension due to the load on the spring.

The necessary operations and formulae are given in Ch. VI., §§ 48-53.

Exp. 50.—Repeat Exp. 46, using now an indiarubber cord. Make two loops at the end of the cord (Fig. 71) and push two pins, A and B, through the cord about an inch away from the loops. Hang a scale-pan from the lower loop, and before starting the experiment proper add a weight sufficient to stretch the cord straight. Note the scale readings of both A and B (call the difference between these the initial length). Then add weights as before; read both A and B each time, and deduce the extension of the portion of cord AB for each load. Plot a graph as before. The result will be the same (if the loads put on are not too great), showing that for small loads the extension is roughly proportional to the load.

Fig. 71.

104. Hooke's Law.—The relation connecting the extension with the load may be looked on as an example of Hooke's Law. This law—enunciated by Hooke in the year 1678 in the form "ut tensio, sic vis"—states that stretching is proportional to the force producing it. The law is true for many cases besides the stretching of a spring or a piece of elastic. Thus the bending of a rod, the deflection of a stretched fiddle-string, the compression of a gas are all proportional to the forces causing them, provided that the strains produced are small.

105. The Bending of a Rod.—A simple apparatus for checking Hooke's Law in the case of the bending of a rod is shown in Fig. 72. A rod AB is mounted on two strong knife-edges, $K\,K$, placed a suitable distance apart. Standing

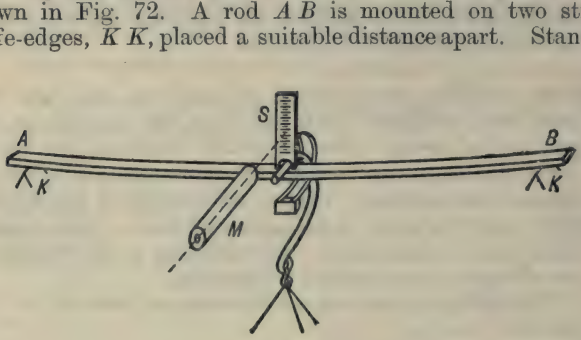


Fig. 72.

on the mid point of the rod is a small millimetre or half-millimetre scale, S, which is kept in position by a suitable balancing attachment as shown in the figure. The weights are placed in a scale-pan hanging from a knife-edge which rests on the rod in a place provided for it immediately underneath the scale. The scale is read by a small low-power microscope, M, furnished with cross-wires, fixed horizontally in a suitable position.

Exp. 51.—To perform the experiment the rod is placed symmetrically on the knife-edges, the scale and the knife-edge carrying the scale are placed in position, and readings of the scale are taken as the load in the pan is increased by equal increments up to a suitable load (not sufficient to bend the rod permanently), and as the load is decreased by the same steps to zero. The readings should agree very nearly. Calculate the depression of the centre of the rod produced by each load. Plot a graph between depression and load. The result will be a straight line showing that the depression is proportional to the load, and thus verifying Hooke's Law.

Capillarity.

106. Behaviour of small quantities of Liquid.—If a small quantity of liquid is placed on a smooth horizontal surface, we might expect it to spread out in a thin film of uniform thickness: this result would seem to be a direct and necessary consequence of gravitation. If experiments are actually carried out the results will be found to differ with the nature of the surface and the liquid. Thus a little drop of paraffin oil allowed to fall on the surface of still water spreads out in a film which may be sufficiently thin to show the colours of the rainbow. Water on polished glass behaves in a similar way. With mercury on glass, however, we obtain different results, for the mercury instead of spreading out gathers up into a pool which is circular and perhaps several millimetres deep. If it is divided up into parts it will be noticed that each is circular in outline, and that if two of them come into contact they unite together, and again form a circular pool. Instead of glass we may use almost any non-metallic surface—wood, paper, etc.—and obtain similar results with mercury. This is not an effect peculiar to mercury: a drop of water will not spread over a greasy plate, but gathers up like mercury on glass.

It is well to notice here that the liquids only spread out indefinitely provided that they are capable of wetting the surface on which they are spread. In other cases the liquids collect in drops or pools. A study of these results suggests that the behaviour of a liquid is not wholly determined by its weight, but that there are other forces in action which are dependent on the natures of the surfaces in contact. These forces are called *surface tensions*, and before discussing them it will be well to call to mind a few familiar facts and describe some simple experiments.

107. Shape of Liquid Drops.—A drop of liquid is always perfectly spherical provided it is not forced to take up any other shape by external forces. This property is utilised in the manufacture of small shot. The molten metal is poured down in a fine stream from the top of a high tower. The stream breaks up into a succession of small drops which cool and solidify as they fall through the air.

To prevent them being knocked out of shape when they reach the ground their fall is broken by a deep bath of water in which they are caught.

The rainbow affords proof that drops of water left to themselves take up the spherical form. The explanation of the shape and colouring of rainbows and haloes is based on the assumption that raindrops are spherical. Any slight deviation from the shape would be sufficient to change the whole character of the phenomena.

The effects of gravity on the shape of the drops in the above illustrations are almost absent because the drops fall freely. If the drops fell through a viscous or dense medium their forms would be altered.

108. Liquid Skins: Soap-bubbles.—A soap bubble is simply a closed film of soapy water filled with slightly compressed air. It is well known that a soap bubble is always spherical. Here we have two opposing sets of forces to be considered, the one the tensions or pulls in the film, the other the forces due to the pressure exerted by the contained air. In accordance with the former the bag assumes the least possible surface area, and in accordance with the latter its volume is as large as possible. Since the shape assumed is spherical we are led to the conclusion that the sphere is a figure which for a given volume has the minimum surface area. This conclusion can be proved mathematically.

Now consider the case of a spherical elastic bag filled with a heavy liquid and resting on a table. If the liquid is under pressure the forces acting are similar to those indicated above, but a third force, gravity, now comes into operation and produces important effects. The bag is no longer spherical; the portion resting on the table is flat and the curvature of the top decreased: a horizontal section through any point is circular, but a vertical section through any point is not circular.

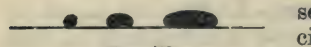


Fig. 73.

Its shape is so similar to that of a drop of mercury (Fig. 73) resting on a horizontal plate of glass that it is reasonable to assume that the forces which mould the drop are similar to the forces regulating the shape of the liquid in the bag. In

other words the behaviour of a drop of liquid is explicable if we regard it as being covered with a thin tightly stretched elastic skin. The tension of this skin or film is called the surface tension of the liquid. It is due to the tension of this skin that the hairs of a wetted paint brush hold together when the brush is removed from the water.

109. Plane Soap Film.

Exp. 52.—Take a piece of wire and bend it into the shape of a ring with a handle; take a thin cotton thread and tie it loosely across the ring. If the ring is now dipped into a soap solution and removed a thin plane film will be obtained divided irregularly into two parts by the cotton. Break one portion of the film—this can be done successfully by touching it with a hot wire—and the tension of the remaining portion will pull the cotton into the form of *an arc of a circle* (Fig. 74).

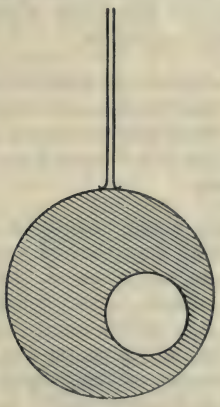


Fig. 74.

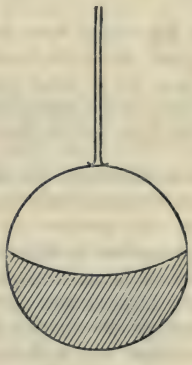


Fig. 75.

Exp. 53.—Again take a fibre of cotton and tie the ends together. Dip the ring of wire into the soap solution and lay the loop of cotton on the film obtained. Break the film inside the loop. The cotton will be pulled out into the shape of a *circle* (Fig. 75).

A circle is the plane figure which for a given perimeter has the maximum area; the soap film takes the form which has the least possible area; the space unoccupied must therefore be a maximum, and the cotton must therefore be pulled out into a circle.

110. Surface Tension in Different Liquids. Tears of Wine.

Exp. 54.—Pour into a flat glass just sufficient water to cover the bottom. Let a drop of ether or alcohol fall into the middle. The water will retreat from the ether and heap itself up around the edges of the dish, leaving the ether in the middle. The motion of the water may be rendered more conspicuous by colouring it or sprinkling a little dust—lycopodium powder—over the surface.

The reason for this motion is that the surface tension of ether is much less than that of water: the water skin is stronger than the ether skin. Hence the ether surface is stretched out, while that of the water contracts.

Exp. 55.—Pour some port wine or other strong wine into a wine glass, shake the liquid round so that the sides are wetted, and then let it stand for a while. Note that the liquid on the sides of the glass gathers itself together in drops and runs down the sides of the glass just like tears.

The explanation of this is usually given as follows:—The surface tension of alcohol is much less than that of water, so that as the alcohol in the wine on the sides of the glass vessel evaporates, leaving the less volatile water behind, the the surface tension is greater than that of the unaltered wine below. This causes more wine to be pulled up on the sides, from which the alcohol again evaporates, leaving water which accumulates till a drop is formed sufficiently heavy to break away and fall as a tear.

The motion of camphor on water is due to this cause.

Exp. 56.—Take a tray of water and sprinkle a little lycopodium powder on its surface. Now sprinkle camphor on the water. Observe the gyratory motions. The motions cease when all the water surface has been contaminated.

Exp. 57.—Take a small wooden boat and fasten a piece of camphor to the stern so that it will just dip into the water. Place the boat on the water, release it and it moves forward.

The reason is that the bows are pulled forward with a force due to the surface tension of pure water, while the drag back on the stern is due to a surface contaminated with camphor. As the surface tension of water is greater than that of a camphor solution the boat travels forward.

111. Definition of Surface Tension.—Consider any small rectangular element of a surface that is in equilibrium. Call it *ABCD* (Fig. 76). The surface forces acting on the edge *AB* may be resolved into two, one acting along the edge and one at right angles to the edge.* Now if *ABCD* is in the surface of a liquid the former of these forces cannot exist, and the only force acting on the edge *AB* is at right angles to it. In the same way, if any line be taken in the surface of a liquid at rest the only forces acting across that line are everywhere at right angles to that line and in the plane of the surface. The total force acting across such a line divided by the measure of its length—or in other words the force per unit length—is called the *surface tension* of this surface. In formulae it is usually denoted by *T*.

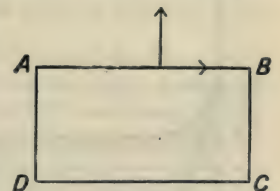


Fig. 76.

The magnitude of the surface tension of any surface depends on the nature of the substances (or *media*) separated by the surface. Thus across each cm. length of a line drawn on a surface separating water and air there is an acting force equal to the weight of ·073 gram; and thus we say that the surface tension of a water-air surface is given by $T = \cdot 073$ gram-wt. per cm.

* Just as an inclined force may be considered constituted of a horizontal and a vertical force.

112. Angle of Contact.—Place some water in a clean glass beaker or trough. Shake the beaker so as to get its sides wet, and then examine carefully the shape of the surface near the sides. It will be noticed that the water surface is curved upwards, as shown in Fig. 77.

Now if O is the upper limit of the water surface there are three surfaces meeting at O, viz. the water-air surface, the water-glass surface, and the air-glass surface.

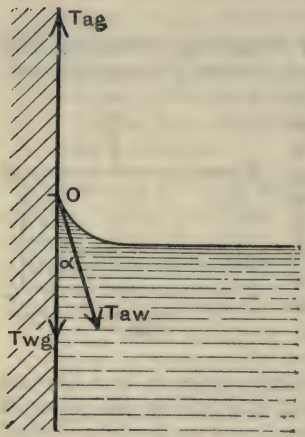


Fig. 77.

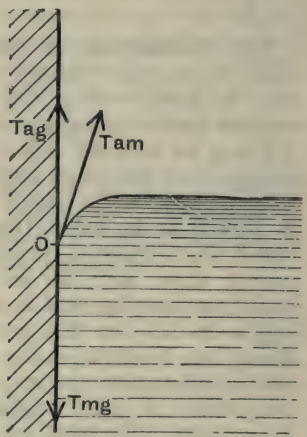


Fig. 78.

Take a unit length of horizontal water-edge through O and let the inclination of the water surface to the vertical at O be α. There are three surface tensions acting, viz.

(1) The surface tension of the air-glass surface, *Tag*, acting vertically upwards.

(2) The surface tension of the water-glass surface, *Twg*, acting vertically downwards.

(3) The surface tension of the air-water surface, *Taw*, acting obliquely downwards at an inclination α with the vertical.

Without going into details it may be remarked that *Tag* is nearly equal to the sum of *Twg* and *Taw*, and remembering that two small forces can only hold a large one in equilibrium if they combine to oppose it, it follows that *Twg* and *Taw* must act nearly in the same direction and opposed to *Twg*. The effect therefore of the three surface tensional forces is to pull the water up at *O*, as in Fig. 77. The angle α between the glass surface and the water surface at the point of contact is called the angle of contact. For water it is very small, very nearly 0° if the glass surface is clean.

The surface of mercury in a trough takes the form shown in Fig. 78. In this case *Tmg* is nearly equal to the sum of *Tag* and *Tam*, and consequently the liquid surface is drawn downwards at the surface of the glass plate so that the angle of contact is greater than 90°.

The resultant upward force per unit length at the side of the vessel in the case of water and the resultant downward force per unit length in the case of mercury are often loosely called the surface tensions of water and mercury respectively.

113. Capillary Tubes.—The elevation of water and the depression of mercury at the sides of a glass trough are small because of the relative insignificance of the surface tension forces in comparison with the weight of the liquid they would have to move. If we can reduce the latter and relatively increase the former the phenomena will be much more marked, and hence these elevations and depressions are best seen when glass tubes of narrow bore are used, and the narrower the tubes the more marked are the phenomena. Very narrow tubes are called capillary tubes (from Lat. *capillus*, a hair): hence the name *capillarity* by which these surface-tension phenomena are often known.

If a series of open glass tubes of gradually decreasing bore (Fig. 79) are immersed vertically in water the water rises in the tubes,* and the narrower the tube the higher it rises. It will also be noticed that the top of the water column is not flat but concave upwards, being curved like the crescent moon in the wider tubes and nearly hemispherical in the narrower

* It is best to suck the water some way up the tube first so as to wet the sides and then let it drop to its equilibrium position.

tubes. This curved surface is called the **meniscus**; in the case of water the meniscus is concave. This curvature is, of

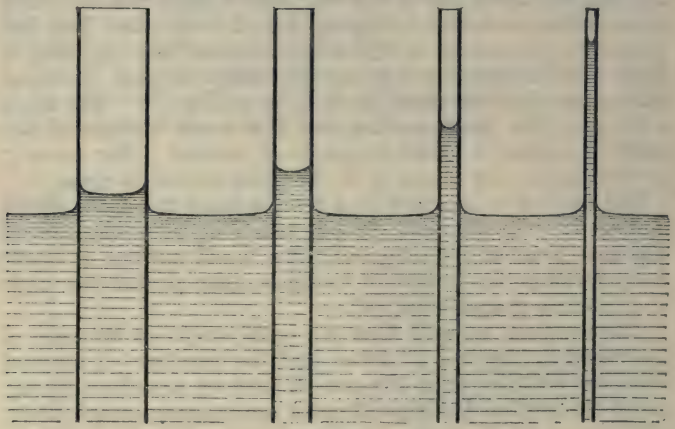


Fig. 79.

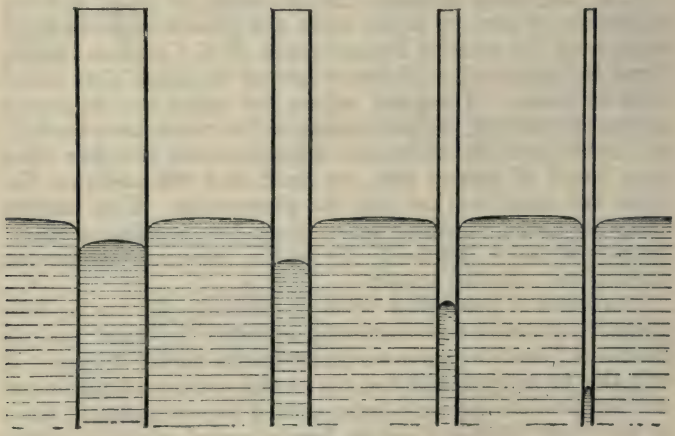


Fig. 80.

course, produced by the surface tensions, as described in § 112.

If the same series of tubes are now dried and immersed in

mercury it will be observed that the mercury is depressed within the tube and the narrower the tube the greater the depression (Fig. 80). In order to see the depression the tubes should be brought close to the side of the trough. The mercury surface is also seen to be convex upwards, the curvature depending on the bore of the tube just as in the case of water. The relation between the height of ascent of water or the depth of depression of mercury in a capillary tube is given by **Jurin's Law**.

For the same liquid and material of tube and at the same temperature the height of ascent (or depth of depression) is inversely proportional to the diameter of the tube.

114. Proof of Jurin's Law.—We shall prove this mathematically in the case of a liquid such as water placed in glass tubes. The tube is wetted; hence the angle of contact is practically zero (Fig. 81). At the top of the column the surface tension forces therefore act vertically around the sides of the tube. The length of the liquid edge is equal to the circumference of the bore of the tube, *i.e.* equal to πd, say, where d = the diameter of the bore of the tube; hence the total upward force

$$= T\pi d \quad . \quad . \quad . \quad . \quad (1)$$

where T = surface tension of the liquid in grams-wt. per cm.

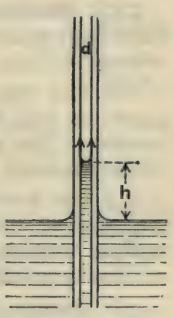


Fig. 81.

Assuming that the tube is cylindrical, let h = mean height of elevation of the liquid column: then the weight of the liquid column = volume of liquid multiplied by its density, ρ (§ 41),

$$= \frac{\pi d^2}{4} . \; h . \rho \quad . \quad . \quad (2)$$

Hence equating (1) and (2) we get

$$T\pi d = \pi \frac{d^2}{4} h\rho$$

or

$$h = \frac{4T}{d\rho}.$$

Hence since for the same liquid in tubes of the same material T and ρ are the same

$$h \propto \frac{1}{d}$$

i.e.

$$hd = \text{a constant.}$$

Thus if a liquid rises 10 cms. in a tube of ·1 mm. bore it will rise 5 cms. in a tube of ·2 mm. bore and 1 cm. in a tube of 1 mm. bore.

The ascent of water in a capillary tube affords a convenient method of finding the surface tension of water.

For since $$h = \frac{4T}{d\rho}$$

$$T = \frac{hd\rho}{4}.$$

For water $\rho = 1$ gm. per c.c.

$$\therefore \quad T = \frac{hd}{4}.$$

Example.—Water rises 14·4 mms. in a tube of 2 mm. bore. Find T.

$$T = \frac{1{\cdot}44 \times {\cdot}2}{4}$$

$$= {\cdot}072 \text{ gm.-wt. per cm.}$$

115. Various Capillary Phenomena.—Some of these have been mentioned on previous pages. We mention a few others:—

(1) The rise of water in narrow cracks, through the pores of soil, blotting-paper, sugar, etc., is due to surface tension. The rise of sap in trees is also largely due to surface tension, for the tubes in trees have very small bores.

(2) The possibility of the formation of bubbles is a consequence of surface tension. Soap solution has a very large surface tension, hence much larger bubbles can be blown with soap solution than with pure water.

(3) Insects run about on the surface of the water. This is because their weight is so small that the minute depressions which they produce on the surface of the water are not enough to break through the water skin.

Similarly a sewing needle, if laid gently on the surface of water, will remain there. The experiment is easily performed if the needle is oily; if, however, the needle is scrupulously clean, and has not come into contact with the fingers, it is at once wetted by the water and sinks.

(4) Two small bodies floating in water, say two small pieces of wood, attract one another if near enough for their menisci to be continuous with each other; similarly bubbles in a cup of tea gather round the edges or group themselves

up in little bunches. If two small pieces of wood are floating on mercury and their menisci come into contact they repel each other.

Summary.—Chapter XI.

1. The ultimate particles or molecules of a body are in rapid motion. By their collisions they tend to separate from one another. By their force of attraction for one another they tend to keep together. Relative importance of these two tendencies in the cases of a solid, liquid, and a gas. (§ 98-99.)

2. The three aspects of the forces between particles: cohesion, adhesion, chemical affinity. (§ 100.)

3. Extension of a spiral string and a rubber cord. (§ 103.)

4. Hooke's Law "ut tensio sic vis," and its illustrations in the case of a spiral spring, an indiarubber cord, and a bending beam. (§§ 103-105.)

5. A liquid behaves as if its surface was a stretched elastic skin. The forces in this skin are called surface tensions. (§ 106.)

6. A drop of liquid if removed from the action of all external forces assumes the shape of a sphere, the sphere being that shape which has the least surface for a given volume. (§ 107.)

7. Soap Film Phenomena. Tears of wine. (§§ 108-110.)

8. Quantitative definition of Surface Tension:—It is the force which acts across a line 1 cm. long drawn on the surface of the liquid. (§ 111.)

9. Angle of contact:—For glass dipping into water the angle of contact is very small, for glass in mercury it is obtuse, being about 127°. (§ 112.)

10. Water and some other liquids rise up some narrow tubes dipped into them, and the rise is greater the narrower the tube. The water meniscus is *concave* upwards. If narrow tubes are dipped into mercury contained in a dish the column of mercury in the narrow tube is below that of the mercury in the dish, and the depression is greater the narrower the tube. The mercury meniscus is *convex* upwards. (§ 113.)

11. Proof of *Jurin's Law*, namely, that the height to which water rises in a capillary tube is inversely proportional to the diameter of the bore of the tube. (§ 114.)

12. Various interesting capillary phenomena. (§ 115.)

EXERCISES XIV.

1. What is the effect of temperature upon the velocity of the particles of a body? Describe what happens to the particles of a liquid when it is heated to boiling and vaporised.

2. What is the difference between cohesion and adhesion? Give an example of each.

3. Define the tenacity of a substance; how would you find the tenacity of brass?

4. What is the meaning of elasticity? Do liquids possess elasticity of shape? If not, why not?

5. If a weight of 20 gms. pulls the extremity of a spiral spring down 1 cm., how far will a weight of 30 gms. pull it down?

6. How would you graduate a spring balance if you were only given one weight that you could attach to it?

7. The lower extremity of a spiral spring is depressed 10 cms. when a lump of quartz is attached, the quartz hanging freely in the air. When a beaker of water is held up so that the quartz is immersed in the water the depression decreases to 6 cms. Find the specific gravity of quartz.

8. A lump of cannel coal (sp. gr. 1·2) depresses the end of a spiral spring 6 cms. when the coal hangs in air. Find the depression of the end of the spring when a trough of water is held up so that the coal is immersed in the water.

9. The following readings on the bending of a beam (see § 105) were taken in the order given.

Load.	Reading of Scale.
0 gms.	2·14 mm.
10 ,,	2·83 ,,
20 ,,	3·50 ,,
30 ,,	4·17 ,,
20 ,,	3·49 ,,
10 ,,	2·82 ,,
0 ,,	2·14 ,,

Plot a curve showing the relation between the load and the depression, and find the depression produced per gm.

10. Describe some experiments which support the analogy that the surfaces of liquids behave like elastic skins.

11. Describe some experiments illustrating the properties of soap films.

12. When tiny pieces of camphor are strewn on the surface of water they rush about hither and thither. Why is this?

13. Define surface tension. What phenomena would lead you to suspect that the surface tension of a soap solution is greater than the surface tension of water?

14. Explain why water rises up a narrow tube while mercury is depressed. Do some experiments with other liquids to find out which behave like water and which like mercury.

15. How would you set out to prove experimentally that the height of ascent of water in a capillary tube was inversely proportional to the radius of the tube? What method would you use for measuring the radius of such narrow tubes?

16. How would you measure the height to which water rises in a capillary tube? What other measurement would you have to make before you could calculate the surface tension of water?

17. A greasy needle can be made to float on water, but an absolutely clean one will not do this. Explain why.

18. Explain why attraction ensues between small particles floating on the surface of water if they come within a certain range of each other.

CHAPTER XII.

TIME. THE SIMPLE PENDULUM.

116. The Unit of Time.—We cannot separate the ideas of time and movement. Time is measured by movements that are known to be regular and uniform. From earliest days the apparent movement of the Sun round the Earth has been used for measuring time; for, although day and night vary in length at different seasons and in different places, it was thought that the interval taken by the Sun in moving from its highest altitude on one day to its highest altitude on the next day was always the same. Later it was found, by observation of the stars, that this interval, called the *solar day*, was not quite constant throughout the year. Therefore the average interval, called the *mean solar day*, is taken as the standard for comparison. This is divided into 24 equal parts, called hours, each hour into 60 equal parts, called minutes, and each minute into 60 equal parts, called seconds. Thus the second is $\frac{1}{86400}$ part of a mean solar day.

117. Methods of Measuring Time.—To be able to divide the day into equal intervals we must observe some change that goes on regularly during the day, and that can be measured. It took many hundred of years to discover such a change. The movement of a shadow cast by the Sun was tried as a time-measurer. But this sun-dial could be used only while the Sun was shining. To be able to measure time at any point of the day or night the flow of liquids was used. Water-clocks were devised in which water flowed continually from a reservoir, and the duration of time was taken as proportional to the amount of water that flowed out during that interval.

SECTION II.—MECHANICS

CHAPTER I.

FORCE AND WEIGHT. MOMENTS.

1. Mechanics defined.— Branches of Mechanics.— The name **Mechanics** was originally used to designate the science of making machines. It is now, however, very generally applied to the whole theory which deals with motion and with bodies acted on by forces.

The subject Mechanics is generally divided into two parts—

(1) **Statics,** which treats of bodies kept at rest under the action of forces;

(2) **Dynamics,** which treats of moving bodies.

2. Particle. — DEFINITION. — *A* **particle** *is a portion of matter whose volume is so small that we can altogether disregard its form, and consider it merely as a portion of matter collected at a single point.*

3. Rest and Motion. — DEFINITION. — A particle that during a certain interval occupies the same position in space is said to be **at rest** during that interval; if at one instant it occupies a certain position, and at another instant a different position, it is said to be **in motion.** Motion is **therefore** equivalent to **change of position.**

4. Force.— DEFINITION. — Force is that which changes, or tends to change, a body's state of rest or motion.

The words *tend to change* are necessary; for the force may not actually cause any change, as its effect may be neutralised by one or more other forces acting on the body at the same time.

The above is the usual definition of force; but the idea of force is really fundamental, and cannot be satisfactorily defined.

When we push against a body or pull at it we exert force on it. If we support a lump of iron on the outstretched hand, it requires muscular effort to prevent the iron from dropping to the ground, because the Earth attracts the iron with a force which we call its weight. It is evident that the force which the hand exerts upwards must be equal to the force with which the Earth pulls the iron downwards.

If, instead of supporting the lump of iron by the hand, we place it on a table, the table exerts an upward force equal to that which the hand had previously exerted. In this case the force is supplied by the natural resistance of the wood of the table to being broken or distorted.

The iron may also be suspended by a string. In this case the upward force is that due to the natural resistance of the string to being elongated. The string is said to be in *tension*.

If, instead of a string, a coiled spring, such as that in a spring balance, is used to support the iron, it will be observed that the force of the Earth on the iron elongates the spring to a definite extent. In this case the force exerted by the Earth on the iron is greater than the force which the unstretched spring is able to exert, and equality is not established until the spring has been elongated to a certain extent.

In each of the above cases the lump of iron is said to be **in equilibrium** under the action of two forces—the downward force due to the Earth's attraction, and the equal upward force due to the hand, the table, the string, and the spring respectively. In Statics equal forces are defined as those which, if acting in opposite directions upon the same particle, would keep it in equilibrium.

5. Dynamical Treatment of Force. — When a body originally at rest is acted upon by a force whose effect is not neutralised or modified by other forces the body begins to move, and, if the force continues to act, the motion becomes faster and faster.

If the same force acts successively on bodies of different mass, it will generate more velocity in a smaller mass than it will in the same time in a larger mass. We may define two forces as *equal* if when they are applied to bodies of the same mass for equal intervals of time they impart the same motion or change of motion to these bodies.

That the dynamical definition of force is equivalent to the statical can be seen from the following considerations:—If two equal forces act on the same particle in exactly opposite directions, the motion which one force tends to impart is exactly the reverse of that which the other tends to impart. The particle cannot move in opposite directions at the same time, and there is no reason why it should move in one direction rather than the other. Hence it will remain at rest, and the two forces will be said to *balance*, or be *in equilibrium*.

The dynamical qualities of force cannot be considered until the properties of motion have been investigated. In the first three chapters we shall deal with forces in equilibrium.

6. Mass and Weight. — The terms **mass** and **weight** have been explained in Chap. IV. of the Introductory Section. We have there defined the *mass* of a body as the *quantity of matter* in the body and the *weight* of the body on the surface of the Earth as the *force with which the body is attracted to the Earth*. It was also stated that weight is proportional to mass.

If we keep to one kind of matter—say iron—we may say that both mass and weight are proportional to volume: *e.g.* 2 cubic inches of iron have twice the mass of, and will weigh twice as heavy as, 1 cubic inch of iron.

We may thus compare the weights of different lumps of iron by simply measuring their volumes.

If, however, we wish to compare the weight of a lump of iron with the weight of a lump of coal, we are met by a

difficulty. The kind of matter in each is different. But the difficulty may be solved by the use of the Ordinary Balance and the Spring Balance.

7. The Spring Balance or Dynamometer.—The ordinary balance and the spring balance have been described in Chap. IV. of the Introductory Section. The principle of the ordinary balance was also dealt with in Chap. IX.; that of the spring balance will be understood better after performing the following experiment:—

Exp. 1.—*To find how the extension of a spring depends upon the load which it supports.* Obtain a long spiral spring. Drive a nail into a stout wooden rod and support the rod vertically in a clamp and stand (Fig. 1). Hang a boxwood millimetre scale from the nail and then the spring in front of it. The lower end of the spring, after forming a loop, is twisted back and finally turned to form an index p pointing to the scale divisions; or the end is passed axially through a cork that carries a needle (n) horizontally (Fig. 2). Suspend a scale-pan from the lower loop by a piece of string so that it hangs clear of the support, &c.

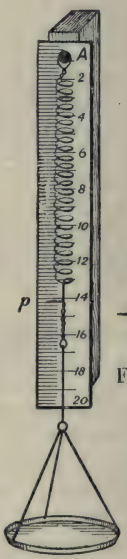


Fig 1.

Fig. 2.

Observe the reading of the pointer, p, when the pan is unloaded, and when loaded successively with 20, 40, 60, &c., grammes, taking readings as the load is increased and also as it is decreased by the same steps. An error will occur in reading p if p is not almost in contact with the scale. If the spring hangs so that p is far from the scale, slightly rotate the spring on its vertical axis until p comes near the scale: then read.

Plot the readings with reference to loads. The graph is practically a straight line showing that the extension is proportional to the load (or force) applied. From the graph deduce the extension per gramme and the load required to produce unit (*i.e.* 1 cm.) extension.

Exp. 2.—*Graduate a spring balance.* Remove the boxwood scale and fix, with drawing pins or small nails, a strip of paper behind the spring. Mark on the paper the position of the pointer for no load and for loads of 10, 20, 30, &c., grammes. Number the lines 0, 10, 20, &c., and subdivide the spaces between into halves or fifths.

We have thus made a scale to the balance, so that when the pointer points to any mark P we know that the load in the scale-pan is P grammes.

Exp. 3.—*Find by the spring balance of the last experiment the weights of coins, &c.* Place the body in the scale-pan; note the number of the division to which the pointer is drawn. Check by weighing on an ordinary balance.

The term **dynamometer** simply means a force-measurer. An ordinary balance measures only a particular kind of force, viz. weights. Since, however, spring balances can be used in any position, they can be used to measure forces in general, *e.g.* if I hold a spring balance in my left hand and extend the spring with the other till the pointer reads one pound, my right hand is exerting on my left hand, and therefore also my left hand on my right hand, a force equal to the weight of one pound.

8. Proportionality of mass and weight. — It is an experimental fact that if two bodies of different masses be allowed to fall *in vacuo* for the same time, they will drop the same distance and have equal velocities at the end of this time. It follows that the forces acting on the bodies, *i.e.* the weights of the two bodies, are proportional to their masses.* The weight of a body is the attraction the earth has for that body. Since the Earth is in rapid rotation about its axis and its shape is not exactly spherical, this force of attraction is not the same at all points on the surface of the Earth, being less at the Equator than at the Poles, and up a high mountain than at the sea-level, *i.e.* the weight of a body is different at different places of the Earth's surface. The mass of a body is, of course, the same all the world over.

* This is fairly obvious at this stage, but is fully explained in § 88.

9. Weighing by a pair of scales and a spring balance.—The variations referred to above in the weight of a body cannot be detected, however, by weighing it with a pair of ordinary scales; for the "weights" that are used are equally affected by the variation of the Earth's attractive force. Whatever change takes place in the force with which the body weighed presses upon the pan, the same change will appear in the force with which the "weights" press upon their pan, and consequently the body and the weights will still balance. Hence **masses, not weights, are compared by a pair of scales.**

With this method of weighing it is important to notice that, if we weigh a pound of sugar at the Equator and another at the North Pole, we should get the same quantity of sugar in each case, *although the attractive force of the Earth is less at the Equator than at the Pole.*

For, if a certain quantity of sugar balances the leaden weight at the Equator, when taken to the Pole both the sugar and the lead will weigh more, and the same quantity of sugar will still balance the same quantity of lead.

With a spring balance, however, the case is different. If a certain force overcomes the elasticity of the spring to such an extent as to depress the pointer 1 in., that force will depress it 1 in. at any other place on the Earth; for the change in the attractive power of the Earth does not affect the elasticity of the spring. When, therefore, we find, as we should, that a body attached to the spring depresses the pointer to different distances at two different places, we infer that the weight of the body has changed. From this we see that **a spring balance compares weights, not masses.**

In this case, if we weigh a pound of sugar at the Equator, we should get more than if we weighed it at the Pole. For, since the sugar weighs less at the Equator, more of it will be required to stretch the spring to a given length than would be required at the Pole.

NOTE.—In practice the variation of the Earth's force of attraction is so small that only the most sensitive spring balance will detect it.

10. Statical Units of force.—The weight of a body is a force, and accordingly the most convenient *units of force* are weights.

In the **English** system the **unit of force** is the weight of a pound and is called a force of **1 pound-weight.** Larger forces may, however, be measured in hundred-weights or ton-weights.

If the **C.G.S.** system is used, the statical unit of force will be the **weight of a gramme** or of a kilogramme (1,000 grammes), according to which is the most convenient.

11. Forces may be represented by straight lines.— When a force acts upon a body, before we can tell what effect that force will have, we must know

(1) **its point of application,** *i.e.* the point of the body at which it is applied; (2) **its line of action,** *i.e.* the straight line in which it would cause a body, acted on by no other forces, to move; (3) **its direction** along the line of action; (4) **its magnitude.**

Now, in the four elements specified above any force can be represented by an arrow-headed straight line for—

(1) We can take a point A on the paper to represent the point of the body at which the force is applied;

(2) From A we can draw a straight line AX parallel to the line along which the force acts. AX will then represent the line of action of the force;

(3) An arrow-head on the line gives the direction of the force along the line of action;

(4) And, lastly, we can cut off from AX a length AB containing as many units of length as the force contains units of force. In this way the magnitude of the force will be represented by the length of AB.

Caution I. — In connection with (4) it is important to notice that the representation may be *on any convenient scale*, provided we keep to that scale throughout the particular problem in hand.

For example, a pound-weight may be represented by a line 1 ft. long, or 1 in. long, or $\frac{1}{10}$ in. long, or any other suitable length. Then a force of 3 pounds-weight, occurring in the same question, *must* be represented by a line 3 ft., 3 ins., or $\frac{3}{10}$ in. long, according to the scale selected.

Caution II. — "The force AB" means a force represented by a length AB in magnitude and acting along AB from A to B, while a force represented by AB in magnitude and acting along AB *from* B *to* A is spoken of as "the force BA." Thus *the order in which the letters are taken indicates the direction of the force* along its line of action.

The force AB is equal and opposite to the force BA.

The graphic representation of forces will be dealt with more fully in Chap. III.

12. Moment of a Force.—If a body is hinged or pivoted at one point and the body is acted on by a force applied at any other point, the only possible motion of the body is one of **rotation** round the first point. It may be, therefore, possible to study the relation between what we may call the rotating effect of a force and its magnitude and position by fixing one point in the body.

DEFINITION. — The **moment of a force about a given point** is **its tendency to produce rotation** about that point regarded as fixed.

Exp. 4.—Bore a clean hole *O* (Fig. 3) through a smooth board. Place it on a smooth table and drive a smooth round nail through the hole at *O*. Support the board on four or five marbles so that it can turn freely round *O*. At any point *A* of the edge of the board attach a spring balance and hold it horizontally so that any force exerted by it would cause motion round *O* in the direction of the hands of a clock. At any other point *B* attach another spring balance so that a force exerted by it would cause rotation in the opposite direction.

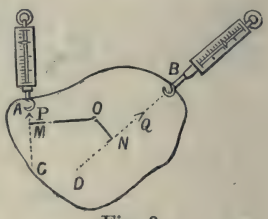


Fig. 3.

Pull the balance out so that the springs are well stretched. Mark the directions *CA*, *DB* of the pulls of the springs on the board.

Draw *OM*, *ON* perpendicular to *CA* and *DB*.

Measure *OM*, *ON* and note the readings of the balances *A* and *B*, giving the forces *P* and *Q* which they exert on the body.

Repeat the experiment several times, varying the positions of the balances. Arrange your results as in the following table:—

VALUES OF *P*.	VALUES OF *OM*.	PRODUCTS OF *P* AND *OM*.	VALUES OF *Q*.	VALUES OF *ON*.	PRODUCTS OF *Q* AND *ON*.

Observation.—Each result in the third column is equal to the corresponding result in the last column.

Deduction. — Since the body is at rest, we conclude that the tendency of the force P to rotate the body in one direction is exactly balanced by the tendency of Q to rotate the body in the contrary direction, *i.e.* the two tendencies are equal in magnitude, that is, their moments about O are equal; and, furthermore, we conclude that **the moment of a force should be measured by the product of the force and the perpendicular from the centre of rotation on its line of action.**

Cor. The moment of a force P about a point O is zero if
either (i.) the force P is zero,
or (ii.) the line OM is zero, in which case O lies on the line of action of P.

Conversely, If the moment of a force P about O is zero, it follows that either (i.) P is zero,
or (ii.) O lies on the line of action of P.

Exp. 5.—Drive a nail A (Fig. 4) into a wall. Draw AB, a vertical, through A. Tie one end of a piece of string to A and the other end to a metal ring C. Also attach to C a string bearing a weight W, say a 7-lb. wt., and a string attached to one hook of a spring balance S reading up to 28 lbs.; the finer the graduation the better. Fasten a piece of cartridge paper against the wall. When the string AC hangs vertical, the reading of the balance is zero. Now, holding the other hook of the balance, gradually pull the balance to one side and, keeping the string CS horizontal, observe that as the angle CAB increases, the reading of the balance increases. To take definite readings, pull out S till AC makes an angle of, say, 10° with the vertical. Arrange the string CS accurately horizontal, mark the centre of the ring C with a pencil dot, and read S.

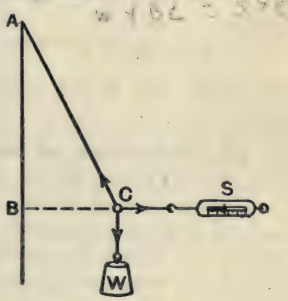


Fig. 4.

The three forces which determine the position of AC are: (1) the vertical force of gravity on the mass W—this is called the weight of W; (2) the horizontal force due to the extension of the spring balance S; and (3) the pull of the string which acts along CA.

Draw CB perpendicular to AB and measure AB and AC. Show that

$$W \times BC = S \times AB,$$

i.e. that the moment of the weight or force W about the point A is equal and opposite to the moment of the force registered by S about the point A. (The moment of the pull of the string about A is zero, for the pull acts through A.)

Repeat for different inclinations of AC, reading S and measuring the lines corresponding to AB and AC. Show that in each case the above relation holds.

The same relation may be obtained by an application of the Triangle of Forces. (*See* Chap. III.)

Note that when AC is nearing the horizontal the force registered by S becomes very large and finally too big for a spring balance to measure. Using the principle of moments, show that whatever may be the weight W the force required to keep AC nearly horizontal is almost infinite.

The experiment could be performed without keeping the string horizontal. In this case a perpendicular AD would have to be drawn up on the line of action CS, and we should get

$$W \times BC = S \times AD.$$

Keeping CS horizontal obviates the trouble of drawing the line AD.

From the above experiments we deduce the following principle :—

The Principle of Moments.—When a body, acted on by several forces in one plane, is in equilibrium, the sum of the moments of forces tending to turn the body one way about any point in that plane is equal to the sum of the moments about the same point of the forces tending to turn the body the other way round.

The Principle of Moments is of great use, especially when proving the relations between the forces of the simple machines. (See *Introductory Section*, Chap. IX., on "Levers.")

Summary.—Chapter I.

1. Force is that which changes, or tends to change, a body's state of rest or motion. (§ 4.)

2. Equal forces are defined—in Statics—by saying that, if they act in opposite directions upon a particle, the particle remains in equilibrium. (§ 4.)

3. Equal forces are defined—in Dynamics—by saying that, if they act for the same time upon equal masses originally at rest, they will produce in each the same motion. (§ 5.)

4. A spring balance or dynamometer measures force. It is usually employed to measure the particular kind of force known as weight. (§ 7.)

5. The *mass* of a body is the same all the world over, but the *weight* of a body is subject to small variations as the body is moved about to various places. (§ 8.)

6. The most convenient units of force are weights: *e.g.* the pound-weight, the gramme-weight, &c. (§ 10.)

7. Forces may be represented in magnitude and direction by straight lines. (§ 11.)

8. The tendency of a force to produce rotation about a point regarded as fixed is measured by the product of the force and its perpendicular distance from the point. To this product is given the name *moment*. (§ 12.)

9. If a body have one point fixed and be in equilibrium under the action of two forces, the moments of these forces about the fixed point are equal in magnitude and opposite in direction. This is the *Principle of Moments*. (§ 12.)

EXERCISES I.

1. What is *force?* How do you define equal forces (1) statically, (2) dynamically?

2. Define *mass* and *weight*. Why is the mass of a body the same all the world over, while the weight of a body changes?

3. What does an ordinary balance compare? What does a spring balance measure?

4. Describe an experiment with a spring balance to show that the extension of a spring is proportional to the force applied.

5. Why can a spring balance be used more generally as a measurer of force than an ordinary balance?

6. What are the statical units of force?

7. Describe exactly how a force may be represented by a straight line. A line 3 ins. long pointing to the N.E. represents a force of 12 units. How would you represent a force perpendicular to the above and of magnitude 8 units?

8. Find graphically the moment about a point O of a force of 3 oz. wt. acting at a point A along a line AB, where OA is 7 ins. and the angle OAB equals (i.) 90°, (ii.) 120°, (iii.) 60°, (iv.) 150°.

9. The moment of a force of 5 units about a given point is $4\frac{2}{7}$ units of moment. What is the distance of the point from the line of action of the force?

10. *ABC* is a triangle having a right angle at *C*; *BC* is 12 ft. and *AC* is 20 ft.; *D* is a point in the hypotenuse *AB* such that *AD* is one-fourth of *AB*. A force of 50 lbs. acts from *C* to *B*, and one of 100 lbs. from *C* to *A*. (*a*) Find the moments of the forces with respect to *D*; (*b*) if the point *D* were fixed, in what direction would the forces make the triangle revolve?

11. *ABCD* is a square, and *AC* a diagonal; in *AC* take a point *O* such that *AO* is a third of *AC*; forces of 20, 7, and 5 units act from *A* to *B*, *C* to *B*, and *D* to *C* respectively. If the side of the square is 6 units long, write down the moment about *O* of each force.

12. *ABC* is a triangle whose sides *AB*, *BC*, *CA* are 3, 4, 5 units in length. Forces of 1, 2, 3 units act along *AB*, *BC*, *CA* respectively. Find the moment of each force round the opposite angular point.

13. By means of a spring balance or a weight and a cord passing over a pulley (Fig. 5), apply a horizontal force to a heavy bar hanging vertically downward, adjusting the pulley so as to keep the cord horizontal. The angle through which the bar is turned will be found to depend on the point to which the cord is attached. Show that (1) the further the cord is from the point of suspension the greater will be the displacement, (2) the greater the pull exerted by the spring balance or the weight the more the bar is displaced. From these deduce that the turning effect of the force depends on (1) the magnitude of the force, and (2) the distance, from the fixed point, of its point of application. Plot a curve to show how, for a given point of application, the force applied varies with the displacement. Weigh the rod and show that the moment of the pull about the point of suspension is equal to the moment of the weight of the rod acting at its centre of gravity (see Chap. II., § 25) about the point of suspension.

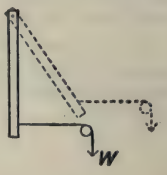


Fig 5.

CHAPTER II.

PARALLEL FORCES AND CENTRE OF GRAVITY.

13. Systems of Forces.—The forces with which we shall deal in *Statics* will always be supposed to be in equilibrium. But it is often necessary to consider the properties of some of the forces apart from the rest. Any number of forces may be called a **system.**

If we have a system of forces acting on a body, we can usually find a single force which could replace the system, *i.e.* which would have the same effect as the given system. This force is called the **resultant** of the system. If we apply to the body a force equal and opposite to the resultant, the body would then be in equilibrium; for the forces acting on the body are then equivalent to two equal and opposite forces (viz., the original resultant and the added force) which would balance one another. This additional force is sometimes called the **equilibrant.**

Conversely, when any number of forces are in equilibrium any one force is equal and opposite to the resultant of all the rest taken together.

14. Equilibrium under two forces.—

Exp. 6. — Obtain a small ring *A* and tie it by strings to two spring balances *S* and *R* (Fig. 6). Fix *S* on a table, and pull *R* until both strings are quite taut. Read the balances *S* and *R*. Draw out *R* further and further. Take frequent simultaneous readings of *S* and *R*.

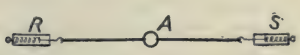


Fig. 6.

Observation.—In every case the readings of S and R are equal, and the two strings are in the same straight line.

Deduction. The ring A is in equilibrium under the two forces exerted by the stretched strings R and S. Hence, if two forces are in equilibrium, they (i.) are equal, (ii.) act in the same straight line, (iii.) act in opposite directions.

15. Parallel Forces.—DEFINITION.—**Forces are said to be parallel when the lines along which they act are parallel.**

When they act in the same direction, they are said to be *like;* when in opposite directions, *unlike.*

For example, if $ABCD$ (Fig 7) is a square, forces in the directions AB and DC are *like* parallel forces; those in the directions AB and CD are *unlike.*

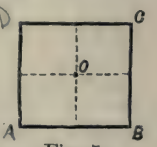


Fig. 7.

16. To find the resultant of two parallel forces.—We make use of the principle that the resultant is equal and opposite to that single force which balances the original forces.

Exp. 7.—*Apparatus.* — (i.) Obtain a rod of fairly soft wood, about 1 m. long, 2 cms. wide, and 1 cm. thick. Graduate the bar in centimetres. At intervals of 5 cms. cut notches into the rod. Prepare little loops of string to just slide along the rod.

(ii.) Obtain three spring balances, measuring up to about 14 lbs. Tie long pieces of string to the rings at the top of the balance.

Adjustments.—Pass the hooks of two of the balances P and Q through loops placed at C and D, C and D being near the ends. Measure CD carefully, and measure an exactly equal distance XY near an edge of the experimenting table (Fig. 8). At X and Y insert two very strong picture rings firmly in the table. Pass the strings of the balances P and Q through the rings at X and Y. Since $CD = XY$, it is possible to arrange the length of the strings and balances so that XY and CD are parallel and the angles at C and D are right angles. When this is done, fasten the strings to the rings.

Pass the hook of the third balance R through a third loop at E on the wooden rod. Note the distance DE. In the straight line YX

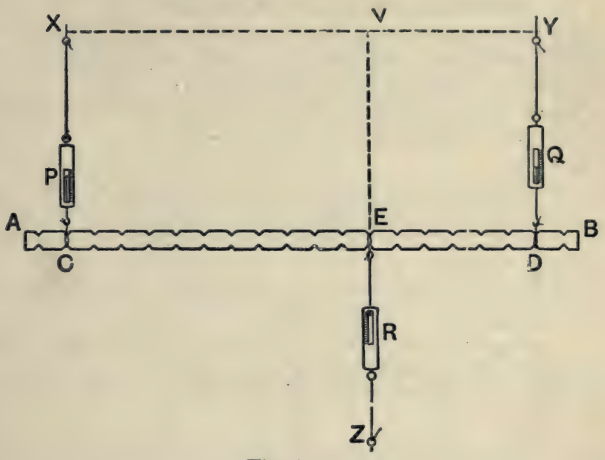


Fig. 8.

measure $YV = DE$; in the line VE produced, mark any point Z, and insert a third strong picture ring in the table at that point. Through this ring pass the string of the balance R until it becomes just taut. In this position there is no tension on the balances.

Procedure.—(i.) Pull out the balance R by drawing the string further through Z. Immediately all three balances indicate tension, and the whole system becomes distorted; so that the reading of the balances tells us nothing about *parallel* forces.

(ii.) Fasten the string at Z tightly. Shorten the string at Y, releasing the string at X as much as may be found necessary, but always keeping the balance P in tension. If this be done slowly and carefully, the bar can be gradually brought into a position parallel to its first position. In this case, since $XY = CD$, and $VY = DE$, the strings ~~XD, YC~~, ZE will be parallel each to the others and perpendicular to the bar AB.

Thus we now have three parallel forces in equilibrium, viz., the three tensions of the spring balances, which act on the rod at the points C, D, E, but produce no motion in the rod. Hence either of these three tensions may be regarded as equal and opposite to the resultant of the other two.

Measurements.—Note the readings of the balances P, Q, R and the distances CD, CE, DE.

The experiment should be repeated, with variations.

(i.) P and Q might be attached to loops other than at C and D; or (ii.) R might be attached to any other loop on AB; or (iii.) the positions of all three balances might be varied simultaneously.

When several sets of readings have been obtained in this way, set them down in tables, as follows :—

(*a*)

P	Q	$P+Q$	R

(*b*)

P	CE	$P \times CE$	Q	DE	$Q \times DE$

(*c*)

P	CD	$P \times CD$	R	DE	$R \times DE$

Magnitude and Direction of the Resultant.—

Observations.—In Table (*a*) notice that corresponding readings in the third and fourth columns are equal, *i.e.*

$$R = P + Q.$$

$$\therefore \quad \text{also} \quad Q = R - P.$$

Deductions. — Now R is the force which balances the two **like** parallel forces P and Q. Hence

The resultant of two **like** *parallel forces is the* **arithmetical** *sum of the forces.*

Also Q is the force which balances the two **unlike** parallel forces R and P. Hence

The resultant of two **unlike** *parallel forces is the* **arithmetical** *difference of the forces.*

In both cases the balancing force opposes the larger of the two given forces, and therefore the resultant acts in the same direction as the larger force.

These three conclusions may be summed up as follows:—

The resultant of any two parallel forces is a third parallel force (1) acting in the direction of the larger, and (2) equal in magnitude to the algebraic sum of the two.

Position of the Resultant.—

Observation.—In Tables (*b*) and (*c*) corresponding values in the third and sixth columns are equal. These are the moments of the forces in each case about the point of application of the resultant.

Deduction.—The moments of two parallel forces about the point of application of their resultant are numerically equal. The experiment shows also that the forces tend to twist the bar in opposite directions about this point; and therefore we may say

The algebraic sum of the moments of two parallel forces about the point of application of their resultant is always zero.

NOTE.—The experiment has been arranged so that all the forces are perpendicular to the bar *AB* because it is easier to make the necessary measurements. But the conclusions deduced are equally true when the forces are inclined at any angle to the bar. A student acquainted with Euclid, Book VI., will easily see that the equation of moments will give the same point in the rod whatever the inclination of the forces.

Hence, in the case both of like parallel forces and of unlike parallel forces, the position of the resultant can be obtained as follows:—

(1) Assume any point as a possible point of application of the resultant.

(2) Calculate the moments of the given forces about the assumed point.

(3) Equate these moments, or make the algebraic sum of the moments equal to zero.

This will give the point of application of the resultant.

COR. I. The resultant of two *equal* like parallel forces acts through a point bisecting the line joining the points of application of the forces.

COR. II. In the case of two equal unlike parallel forces the above method would give a resultant whose magnitude is zero and whose line of action is infinitely distant from the two equal forces. In this case it is not possible to find a single force which will replace the first two, the effect of these two on a body being merely to twist it round without moving it in any direction as a whole.

NOTE.—**Whether the forces are like or unlike, their resultant acts *nearer* to the *greater* force, between two like forces, and beyond the greater of two unlike forces.**

Examples.—(1) The resultant of two like parallel forces of 5 and 3 units acting at points A, B (Fig. 9) where AB is 12 ins. is a force—

(i.) whose magnitude is $(3+5)$ or 8 units

(ii.) whose line of action is parallel to those of the given forces and passes through a point C on AB, such that $5AC = 3BC$ (by moments about C);

(iii.) whose direction is the same as that of the 3 or 5 units.

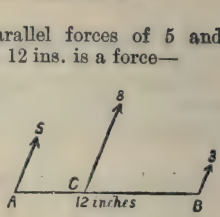


Fig. 9.

Now $AC + BC = 12$, or $BC = 12 - AC$.

Substituting for BC, we get

$$5AC = 3(12 - AC);$$

$$\therefore \quad 8AC = 36, \quad \text{or} \quad AC = \tfrac{36}{8} = 4\tfrac{1}{2} \text{ ins.},$$

i.e. the resultant force acts through a point C in AB, $4\frac{1}{2}$ ins. from A.

(2) The resultant of two unlike parallel forces of 5 lbs. and 4 lbs. wt. acting at points A, B (Fig. 10), where AB is 3 ft., is a force—

(i.) whose magnitude is $(5-4)$ or 1 lb. wt.;

(ii.) whose line of action is parallel to those of the given forces and passes through a point C on AB produced through A, the point of application of the greater force, such that $5AC = 4BC$ (by moments about C);

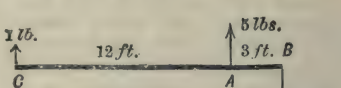


Fig. 10.

(iii.) whose direction is the same as that of the force of 5 lbs. wt.

Now $BC = AC + AB = AC + 3$.

Substituting for BC, we get

$$5AC = 4(AC + 3) = 4AC + 12; \quad \therefore \quad AC = 12,$$

i.e. the resultant force acts through a point C in AB produced through A, 12 ft. from the greater force.

17. Graphic Construction for the Resultant.—

(i.) **Like Forces.**—Along the lines of P and Q (Fig. 11) measure off Aa and Bb respectively proportional to Q and P (*i.e.* the forces reversed). Join Ab and Ba, and let the lines cut in X. Through X draw a straight line parallel to Aa and Bb, cutting AB in C. Measure off along this line Cc equal to the sum of P and Q. This is the resultant R.

Students who have read Euclid, Book VI., will be able to follow the following proof:—

$$\frac{P}{Q} = \frac{Bb}{Aa} = \frac{BX}{Xa} \text{ (VI. 4)}$$

$$= \frac{BC}{AC} \text{ (VI. 2)}$$

$$= \frac{\text{perp. from } C \text{ on } Q}{\text{perp. from } C \text{ on } P} \text{ (VI. 2).}$$

Hence

$P \times$ perp. from C on P
$= Q \times$ perp. from C on Q,

or the moment of P about C = the moment of Q about C.

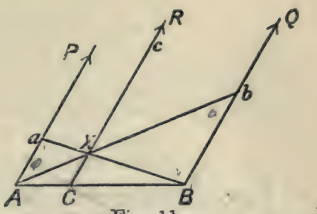


Fig. 11.

(ii.) **Unlike Forces.**—The construction and proof should now be evident to the student from the adjoining figure. (Fig. 12).

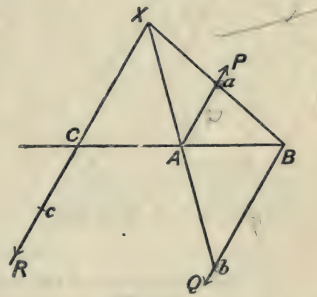


Fig. 12.

18. Resultant of more than Two Parallel Forces.—If three parallel forces (A, B, C) be applied to a bar, A and B could be replaced by their resultant R without changing the mechanical conditions of the bar. In the same way R and C could be replaced by their resultant S, *i.e.* S could replace the three forces A, B, and C.

Similarly, if there were originally a fourth force D, a single force T could be found which would replace the four.

Generally, if there were any system of parallel forces, a single force could be found able to replace them without changing the mechanical conditions of the body.* The magnitude and direction of this resultant are determined from the following considerations:—

(i.) The magnitude is equal to the difference between the sum of all the forces acting in one direction

* We have assumed that it is always possible to find the resultant of two parallel forces. In the case of two unlike parallel forces which are equal, the above figures show that the resultant is zero, and the point X is infinitely distant. In this case it is not possible to find a single force which will replace the first two.

and the sum of all the forces acting in the opposite direction.

(ii.) The line of action is parallel to those of the original forces, and its direction along this line is the same as that of the forces whose sum is the greater.

19. Centre of Parallel Forces.—In finding the position of the resultant of a number of parallel forces, the point of application does not depend upon the inclination of the lines of action of the forces, but only on their magnitudes and the positions of their points of application. *Whatever may be the inclination of the lines of action*, so long as the magnitudes of the forces and their points of application are not altered, the point of application of the resultant is not altered; that is to say, the line of action of the resultant always passes through a certain point. This point is called the **centre of the parallel forces.**

EXERCISES II.

1. Find the magnitude and a point in the line of action of the resultants of the following pairs of *like* parallel forces :—

(i.) 2 units and 1 unit, 3 ft. apart.
(ii.) 3 units and 5 units, 2 ft. apart.
(iii.) $\frac{3}{7}$ lb. wt. and $\frac{1}{14}$ lb. wt., $3\frac{1}{2}$ yds. apart.
(iv.) 23 gms. wt. and 37 gms. wt., 1 yd. apart.

2. Find the magnitude and a point in the line of action of the resultants of the parallel forces in Question 1, if the given forces are *unlike*.

3. Three forces of 2, 10, and 12 units act along parallel lines on a rigid body. Show how they may be adjusted so as to be in equilibrium.

4. The resultant of two like parallel forces, 9 and 11 units, acts along a line 2 ft. 9 ins. from the line of action of the larger force. Find the distance between the lines of action of the forces.

5. Two forces of 10 units each act on a body along parallel lines, and in opposite directions. Show by a diagram that it is impossible to balance these forces by any one force.

6. A rod, 10 ft. long, whose weight may be neglected, has masses of 8 lbs. and 11 lbs. attached, one to each end. Find the point about which it will balance, and the force required to support it.

7. Parallel forces P, Q, R act at points A, B, C in a straight line. Find the magnitude of the resultant and its position when—

(i.) $P = 2$, $Q = 3$, $R = 4$, $AB = 2$, $BC = 3$.
(ii.) $P = 1$, $Q = -2$, $R = 3$, $AB = 3$, $BC = 2$.
(iii.) $P = 2$, $Q = -3$, $R = -2$, $AB = 4$, $BC = 3$.

Centres of Gravity.

20. Gravity.—The Earth attracts each particle of a body with a force, called the *weight* of the particle, directed towards the centre of the Earth and in proportion to the mass of the particle.

Therefore the force exerted by the Earth upon any body of finite size, which we term its *weight*, is the resultant of all the forces exerted by the Earth upon its separate particles.

Now, consider a body whose size is very small compared with that of the Earth. Straight lines drawn from its various points to the centre of the Earth meet at so great a distance away from the body that they may be regarded as parallel. But it is along these lines that the forces exerted by the Earth on the particles of the body act. *These forces may therefore be regarded as parallel.*

Now we cannot alter the direction of the forces due to the attraction of the Earth, for this must always be vertical; but, if we turn the body round, we change the inclination of the lines of action of the forces *relative to any line fixed in the body* without altering their magnitudes or points of application *in the body*. It follows, from § 19, that, in whatever position a body may be placed, *the resultant of all these parallel vertical forces passes through a point fixed relative to the body*. In other words, the resultant of all the weights of the individual particles of which the body is composed passes through a fixed point of the body. This point is called the *centre of gravity* of the body, and this resultant is called the *resultant weight*.

Definition.—**The centre of gravity** (**C.G.**) *of a body is that point* **fixed relative to the body** *through which the line of action of the* **resultant weight passes, whatever** *be the* **position** *of the body.*

Caution.—This definition supposes that in its various positions no change is made in the size or shape of the body. The C.G. of an open book is not the same as the C.G. of the book when closed.

The **resultant weight** is the sum of the weights of its separate particles (see § 18).

We may therefore say that the **C.G. of a body is that point of it at which the whole weight of the body may always be supposed to act.**

Example.—Weights of 4 lbs. and 7 lbs. are placed at the ends of a bar (supposed without weight) 3 ft. long. Find the c.g. of the two weights.

For ease of representation, suppose the weights 4 and 7 to be hung at the ends of the bar.

Let AB (Fig. 13) represent the bar, G the centre of gravity, and W, acting at G, the resultant of the weights.

The position of W is determined by the condition that the algebraic sum of the moments of the forces round G is zero.

Thus $4 \times AG - 7 \times BG = 0$.

But $BG = 3 - AG$.

Substituting for BG,

$$4 \times AG - 7(3 - AG) = 0$$

or $$11 \times AG = 21.$$

$$\therefore \quad AG = \tfrac{21}{11} = 1\tfrac{10}{11}.$$

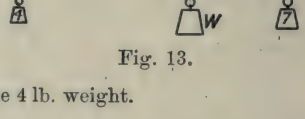


Fig. 13.

Therefore G is situated $1\frac{10}{11}$ ft. from the 4 lb. weight.

EXERCISES II. (*continued*).

8. Find the positions of the c.g.'s of the following systems :—

(i.) 1 lb. and 3 lbs. placed 3 ins. apart.

(ii.) 5 tons and 7 tons placed 16 ft. apart.

(iii.) 20 gms. and 46 gms. placed 11 cms. apart.

9. The mass of the Earth is 39 times that of the Moon. Where is their common c.g., the distance between their centres being 240,000 miles?

21. If a body be supported at its centre of gravity, it will rest in any position.

For, in whatever position the body be placed, the two forces acting on it, viz., its weight and the reaction at the point of support, pass through the point of support, and therefore cannot turn the body round the point of support.

This property of the c.g. gives us an alternative definition :—

Definition.—**The centre of gravity** *of a body is that* **point fixed relative to the body** *which is such that, if the body is supported there, it will rest in any position.*

Observation.—In these definitions note that we do not say the c.g. is a point *in* the body, for in many cases (*e.g.* a hoop) that point lies outside the body. The point is fixed relative to the body in both cases, and in the second definition, if the point is outside the body, it must be supposed to be rigidly attached to it by weightless rigid wires.

22. If a body be at rest when suspended from any point, the centre of gravity and the point of suspension are in the same vertical line.

Let a body be suspended from a point *O* by means of either a hinge at *O* (Fig. 14) or strings attached to *O* (Fig. 15).

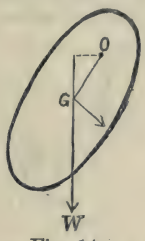


Fig. 14.

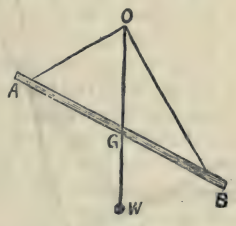


Fig. 15.

Then the only forces acting on the body are

(i.) its weight acting at *G*, its C.G., vertically downwards;

(ii.) the reaction at *O* or the tension of the strings.

All the forces of (ii.) act through *O*, and therefore their resultant acts through *O*.

But, since the body is in equilibrium, the resultant of these forces must be equal and opposite to the weight of the body.

Therefore the resultant of the forces at *O* must act also through *G*, and be vertical, *i.e.* *OG* must be vertical.

Therefore the C.G. and the point of suspension are in the same vertical line.

23. To determine the Centre of Gravity of a body practically.—By means of the property proved in the last section, we are able to experimentally determine the position of the C.G. of a body. We will take as an example the case of a thin flat sheet of metal of any shape.

Exp. 8.—Suspend the sheet of metal from a point of it, *A* (Fig. 16), by means of a string, and from the same support hang a plummet line (a thread tied to a piece of lead). Mark on the surface two points *A* and *D* (as far apart as possible) on the plummet thread. Join *A* and *D* by a straight line.

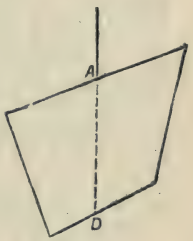


Fig. 16.

Next suspend it from another point *B* (Fig. 17), and on the metal mark the new position of the plummet thread, *BE*.

Deduction.—By § 22, the C.G. must, in Fig. 16, be in the vertical through *A*, *i.e.* in the line *AD*; and similarly, in Fig. 17, it must be in *BE*.

Therefore it is at *G*, their point of intersection, or rather it is behind *G*, halfway through the sheet.

If, further, the body is suspended from any other points, it will in all cases be found that the vertical lines through the points of suspension all pass through *G*, the intersection of any two of them.

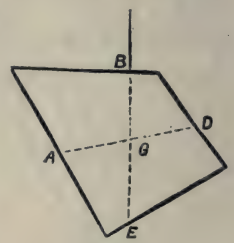


Fig. 17.

The student should test this practically with bodies of various shapes, *e.g.* a piece of cardboard and a piece of bent wire. In the latter case the wire should be fastened to a thin flat sheet of paper on which the verticals may be marked.

The above method is applicable to a large number of solid bodies, *e.g.* a tricycle or a chair. In most cases, however, the form of the body prevents the experiment being carried out. For the continuations of the verticals pass into the material of the body and intersect there, *e.g.* with an irregular log of wood. When the body consists of a mass of *framework*, as in a bicycle, its C.G. can be found by this method.

Exp. 9.—Balance the above sheet of metal on the bevelled edge of a ruler; mark two points *A*, *D* (Fig. 16) at which the edge touches the sheet; draw a line through them. Repeat for another position, obtaining, say, the line *BE* (Fig. 17). *G*, the point of intersection of *AD*, *BE*, is the centre of gravity.

Exp. 10.—Cut out an ellipse from cardboard, find its C.G. (centre of gravity) graphically, and verify by suspending it.

Exp. 11.—Make a cardboard square, divide it into four equal squares, cut away one portion, and find the C.G. of the remainder (*a*) by calculation, (*b*) experimentally.

Exp. 12 Cut out a trapezium and find its C.G. by calculation, regarding it as the difference between two triangles. Verify by suspending it.

Exp. 13.—On a sheet of cardboard draw a circle with radius 14 centimetres. Take a point on the circumference as centre, and with the same radius mark off two points on the circumference. Join these points by a straight line. Find (i.) the area, (ii.) the C.G. of the figure bounded by the straight line and the larger arc.

24. DEFINITIONS.—**Lamina.—Symmetrical Bodies.**

A body is said to be **uniform** or **of uniform density** when a cubic inch of one portion contains as much matter as a cubic inch of any other portion of the body.

A **lamina** may be regarded as an area over which a very thin layer of matter has been spread.

If the matter is spread uniformly, the lamina is said to be *uniform.*

A sheet of notepaper may be regarded as a uniform lamina.

By the C.G. *of an area or surface* is meant the C.G. of the lamina formed by spreading matter uniformly over that area or surface.

A body is said to be **symmetrical** with regard to a point O if it can be divided up into pairs of particles, the particles of each pair being equal, and O being the middle point of the line joining them.

Thus a uniform ruler may be divided up into pairs of equal particles equidistant from the middle point of the ruler, which is therefore symmetrical with regard to its middle point.

Again, a sphere may be divided up into pairs of particles equidistant from the centre. A sphere is therefore symmetrical about its centre.

25. Centres of Gravity of Symmetrical Bodies.—If a body is symmetrical with regard to a point G, that point is the C.G. of the body.

For the C.G. of any two equal particles is at the middle point of the line joining them.

The whole body can be divided up into pairs of equal particles, such that G is the middle point of the line joining each pair.

Therefore G is the C.G. of every pair of particles.

Therefore G is the C.G. of all the particles, *i.e.* of the body.

The following are particular cases of the above theorem:—

The centre of gravity of

a straight line or *uniform rod* is *its middle point;*

a circular ring or *area* is *the centre of the ring* or *area;*

the area or *volume of a sphere* is *the centre of the sphere;*

a square or *parallelogram*	is *the intersection of its diagonals;*
a cylinder	is *the middle point of its axis;*
a cube	is *the intersection of its diagonals.*

In the case of a triangle we may suppose the figure to be made up of a number of thin strips parallel to the base (Fig. 18). The centre of gravity of each strip is at its middle point; therefore the centre of gravity of the triangle lies on the line joining these mid-points, *i.e.* on the median AD.

Fig. 18.

Similarly, it lies on the median CE; therefore it is at G, the point of intersection of these medians, and by simple geometry it may be shown that $AG = \frac{2}{3}AD$.

Exp. 14.—*Find the* C.G. *of a triangle.* Use a cardboard figure and either Exp. 8 or Exp. 9.

Now join the middle point of each side with the opposite corner. Observe that the three lines (medians) intersect at the C.G. Show by actual measurement that the C.G. is one of the points of trisection of each median.

Example.—Two uniform rods, of weights 6 lbs. and 4 lbs. and of equal length, 2 ft., are fastened together at one end at right angles. Find the centre of gravity.

Let AB and BC (Fig. 19) be the rods fastened at B at right angles to each other. Since the rods are uniform, we may imagine the whole weights, 6 lbs. and 4 lbs., to act vertically at D and E. Join DE. Then practically we have two like parallel forces of 6 lbs and 4 lbs. acting at D and E, where $DE^2 = 1^2 + 1^2 = 2$ or $DE = \sqrt{2}$ ft.

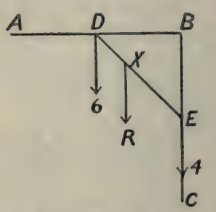


Fig. 19.

Let X be the centre of gravity of the whole system. Then the 6 lbs. at D and the 4 at E are together equivalent to 10 at X. To find X, we remember that (i.) the moments of R at X and 4 lbs. at E about D are equivalent, for 6 lbs. at D produces no moment round it, and (ii.) these moments are proportional to $R \,.\, XD$ and $4 \,.\, DE$.

Thus $$10 \,.\, XD = 4 \,.\, DE = 4\sqrt{2} \text{ ft.}$$

Hence $$XD = \frac{4\sqrt{2}}{10} \text{ ft.} = \cdot 564 \text{ ft.}$$

or the C.G. is in DE and ·564 ft. from D.

26. Stable, Unstable, and Neutral Equilibrium.—From § 22 we learn that a body can be balanced by supporting it at any point in the same vertical line as its C.G., whether that point is *above*, *below*, or *at* the C.G. The first and third cases are practicable, but the second case is frequently not, for it may be found that the least disturbance will cause the body to fall away altogether from its balancing position.

Example.—An egg will remain at rest when laid on its side on a table. It would, however, be difficult to balance it so that it rests on the narrow end, and, even if this has been managed, a breath of air would cause the egg to fall on its side again.

When a body is easily balanced, as is always the case when a point above the C.G., and sometimes when a point below the C.G. (egg on side), is supported, and the body is slightly disturbed, it either returns to its original position or remains in its new position. On the other hand, if it is difficult to balance the body, and the body be disturbed, it will fall completely away from the position in which it was balanced.

We have, then, three kinds of equilibrium—

If a body is at rest in such a position that, if slightly disturbed, it would tend to return to its original position, the body is said to be in stable equilibrium.

Examples.—Ball inside bowl; a right cone resting on its base; pendulum; any body suspended from a point *above* its C.G.

If a body is at rest in such a position that, if slightly disturbed, it would tend to move further from its original position, the body is said to be in unstable equilibrium.

Examples.—Egg balanced on one end; cone balanced on its vertex; stick balanced vertically on the finger; *most* bodies supported *below* C.G.

When the flat sheet of metal of Exp. 9 is balanced on the ruler it is in unstable equilibrium.

If a body is at rest in such a position that, if slightly disturbed, it shows no tendency to return to its original position or to move further from it, the body is said to be in neutral equilibrium.

Examples.—Cone resting on its side; sphere or round ruler lying on table; any body supported *at* its C.G. (see § 21).

Exp. 15.—Find the C.G. of the given |___-shaped piece of wire bent at right angles. Verify by balancing it on a straight edge.

27. Base.—DEFINITION.—If a body rests upon any plane surface, horizontal or inclined, and a fine string be drawn *tightly* round it close to this surface, so as to enclose all the points of contact with the surface, the area enclosed by the string is called the **base** of the body.

Examples.—The base of a three-legged table standing on the floor is the triangle (usually equilateral) formed by joining the three feet.

The base of a four-legged table is the square or rectangle formed by joining the four feet.

Thus the base is in all cases a polygon without any re-entering angles. It is what is ordinarily meant by a polygon in Euclid, each of its angles being less than two right angles.

28. Conditions of Equilibrium of a body resting on a Plane Surface, horizontal or inclined, on which no slipping can take place.

When a body is placed on a plane surface, it will stand or overturn according as the vertical line through its centre of gravity passes within or outside its base.

Let the vertical line through the C.G. G meet the plane surface in M, and let AD represent the base of the body.

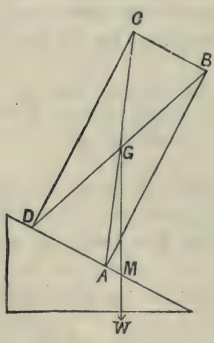


Fig. 20.

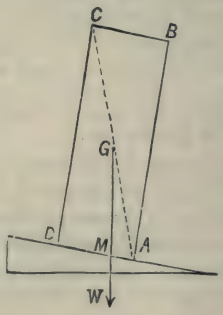


Fig. 21.

The resultant reaction of the plane is a force acting away from the plane, and, therefore, tending to turn the body round A in the "clockwise" direction (in the figure).

In Fig. 20, where the vertical through G passes outside the base, the weight also acts so as to turn the body round A in the "clockwise" direction.

Thus, in this case all the forces acting on the body tend to turn it round A in the same direction. The body will therefore overturn.

In Fig. 21, where the vertical through G passes within the base, the moment of the weight round A is in the "counter-clockwise" direction, and motion in this direction is prevented by the presence of the plane. In this case, therefore, the body will remain at rest.

COR.—**If a plane on which the body is resting is tilted up, the body will overturn as soon as its C.G. is vertically over the boundary of the base.**

Illustrations.—(1) A man carrying a parcel on one arm leans towards the other side so as to keep the common C.G. of himself and parcel vertically over the base formed by his feet. It is much easier to carry two parcels of half the weight, one in each hand; for then the C.G. falls vertically *over* the base, and there is no need to disturb the body to secure this condition.

(2) A man when ascending a hill leans forward, and when descending leans backward. If he has a load to carry, he puts it in front of him when going up hill, and behind him when going down hill.

(3) In Fig. 22 the topmost book would fall, as the vertical through its

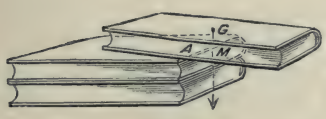


Fig. 22.

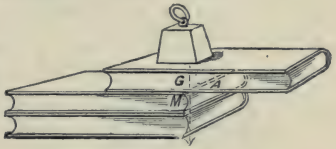


Fig. 23.

C.G. falls beyond the base, *i.e.* the portion of the second book with which it is in contact. It is prevented from doing so by placing a weight on it, Fig. 23, and thus moving the common C.G. of the weight and book further to the left, so that the vertical through it meets the base.

Exp. 16. — By a graphic construction, find the angle at which a square prism will begin to topple over. Verify by experiment.

EXERCISES II. (*continued*).

10. Explain, by the aid of drawings, why a man leans forward in going uphill and backwards in going downhill.

11. A man, with a bucket of water in one hand, stands with his feet close together. Why is it that in order to preserve his balance the man has to stand with his body leaning to one side? Illustrate your answer by a sketch.

12. If you had a short piece of string given you, and a rod heavier at one end than at the other, *e.g.*, a walking stick, how would you find the point in its length at which its C.G. is situated?

13. Mention an experimental way of showing that the C.G. of a circular board is at its centre.

14. If a lamina in the shape of a parallelogram weighs 4 lbs., and a particle weighing 1 lb. is placed at an angular point, where is the C.G. of the whole situated?

Summary.—Chapter II.

1. The resultant of a system of forces acting on a body is that single force which would have the same effect on the body as the system of forces. (§ 13.)

2. The *resultant* of a system of parallel forces

(*a*) Is parallel to the system;

(*b*) Is equal in magnitude to the algebraic sum of the forces;

(*c*) Acts at a point about which the algebraic sum of the moments of the forces is zero. (§ 16.)

3. The centre of gravity of a body is that point fixed relative to the body through which the resultant of all the parallel forces due to the Earth's attraction on the particles of the body passes, whatever be the position of the body. (§ 20.)

4. The centre of gravity of a body has the following properties:—

(*a*) All the weight may be supposed to be concentrated there without altering the effect of the Earth's attraction. (§ 20.)

(*b*) If a support be placed there, the body will rest in any position under the Earth's attraction. (§ 21.)

(*c*) If the body be suspended from any point, the vertical line through the point of suspension will also pass through the centre of gravity. (§ 22.)

(*d*) If the vertical line through the centre of gravity of a body fall outside the base of the body, the body will turn over: otherwise the body will remain steady. (§ 28.)

EXERCISES II. (*concluded*).

15. Draw two parallel straight lines, and suppose forces of 5 and 7 units to act along them respectively towards the same parts. Find their resultant, and show by a diagram exactly how it acts.

16. Two parallel forces of 3 and 4 units act on a body in opposite directions. Specify completely the force required to balance them, and show by a diagram how the three forces act.

17. A and B are two rigidly connected points 5 ft. apart; forces of 5 and 7 units act at A and B respectively at right angles to AB and in the same direction; they are balanced by a force P acting at a point O in AB. Find (*a*) the magnitude of P, (*b*) the distance AO.

18. Two like parallel forces, P and Q, act at two points in a straight line 4 ins. apart. Their resultant is a force of 2 units acting at a point 1 in. from the point of application of P. Find the values of P and Q.

19. What are the values of P and Q in Question 18 if the forces are *unlike?*

20. Forces of 5 and 7 units act in the same direction along parallel lines at points 2 ft. apart. Where is their centre? If the direction of the former force is reversed, where will now be their centre?

21. Two parallel forces of 10 and 12 units act in opposite directions at points 2 ft. apart. Where is their centre?

22. A rod, whose weight can be neglected, rests on two pegs 12 ins. apart; a weight of 10 lbs. hangs on the rod between the points, and 4 ins. from one of them. What are the forces on the pegs?

23. A man carries a bundle at the end of a stick which is placed over his shoulder. If the distance between his hand and his shoulder be changed, what alteration occurs in the force on his shoulder?

24. Describe an experiment to prove that the resultant of two parallel forces is equal to the algebraical sum of the forces.

25. Take a uniform bar 4 ft. long and support it at its centre. Suspend a weight of 2 lbs. at a distance of 18 ins. from one end. What weight must be suspended at the other end to balance the bar?

26. Two men have to carry a weight of 1 cwt. slung on a pole 12 ft. long, each man supporting one end of the pole. If one of the men is twice as strong as the other, where must the weight be slung on the pole that each man may have to carry his fair share of the load?

27. A uniform rod is pivoted at its middle point, and a weight of 20 gms. is attached at a point 25 cms. from the fulcrum. To what point on the rod must a weight of 15 gms. be attached in order that the rod may balance in a horizontal position?

28. Where is the c.g. of a square piece of cardboard situated? If the cardboard weighs 1 oz., and a weight of $\frac{1}{2}$ oz. is placed at one corner, where will the c.g. of the whole be?

29. Two rods of uniform density are put together, so that the one stands on the middle point of the other and at right angles to it; the former weighs 3 lbs. and the latter 2 lbs. Find the c.g. of the whole.

30. If a square tin plate weighs 5 oz., and a small body weighing 2 oz. is placed at one corner, where will the c.g. of the whole be?

31. Two uniform cylinders of the same material, one 8 ins. long and 2 ins. in diameter, the other 6 ins. long and 3 ins. in diameter, are joined together end to end, so that their axes are in the same straight line. Find the c.g. of the combination.

32. A right-angled triangle is suspended by a string from its right angle. Draw a diagram showing the position in which it will hang.

33. A particle is placed at an angular point of a triangular lamina of uniform density; the weights of the particle and lamina are equal. Show, in a diagram, where the c.g. of the whole is situated.

34. Why is a coach laden with passengers on the outside more liable to be upset than when it is laden on the inside?

35. A square sheet of cardboard weighing 8 ozs. is suspended by a thread fastened to one corner, and a weight of 4 oz. is fastened to one of the corners adjacent to the corner of suspension. Draw a diagram to show the position in which the sheet will hang, and say what is the total weight that the thread supports.

CHAPTER III.

THE PARALLELOGRAM OF FORCES.

29. In Chap. II., § 14 we found the conditions that must exist if two forces are in equilibrium: (1) they must be equal; (2) they must act in the same straight line; (3) they must act in opposite directions. Further on in Chap. II. we dealt with the equilibrium of parallel forces. We now deal with forces that are not parallel.

30. Equilibrium of a particle under the action of three forces.

Exp. 17.—Tie two spring balances *P* and *Q* to the ring *A* (Fig. 24), and fasten the balances to a table. Pull at the ring *A*: the balances at once indicate that they are exerting forces on *A*.

In order to determine the force which balances *P* and *Q*, tie a third balance *S* to *A*, and pull it out until *P*, *Q*, *S* are all extended. At this stage *S* balances *P* and *Q*. Read *P*, *Q*, and *S*.

If this procedure be repeated several times, *P*, *Q*, *S* being extended further and further, *S* in every case balances *P* and *Q*.

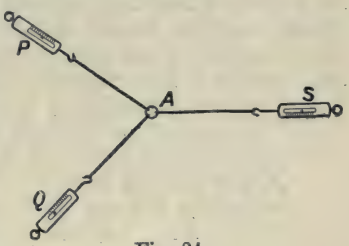


Fig. 24.

Deduction.—Three forces are acting on *A*, namely, those exerted by the stretched strings *P*, *Q*, and *S*; hence this experiment teaches us that whenever two forces are applied to a particle it is always possible to find a third force which balances them.

Observations.—(1) The three strings, if produced, always meet at the centre of the ring.

(2) That the reading on *S* is not greater than the sum of the readings on *P* and *Q*.

31. Two forces acting on a particle can be replaced by a single force.—In Exp. 17 a single force *S* was

balanced by the joint action of two forces, P and Q. In Exp. 6 a single force S was balanced by a single force R. Therefore there would be no disturbance of the ring A if the single force R of Exp. 6 were substituted for the two forces P and Q of Exp. 17. In other words, in such experiments two forces acting on a particle may be *replaced* by a single force without disturbing the state of equilibrium. This single force is called the **resultant** of the two given forces (see § 13).

32. Magnitude and Direction of the Replacing Force.—The replacing force is determined practically (i.) by balancing the two forces against a single force as in Exp. 17, and then (ii.) making use of the fact established in Exp. 6, namely, that the replacing force is equal and opposite to this. The general law establishing the relation of two forces to the single force which may replace them is as follows:—

33. THE PARALLELOGRAM OF FORCES. — If two forces acting on the same particle be represented in direction and magnitude by two adjacent sides of a parallelogram drawn from their point of application, the diagonal of the parallelogram drawn from that point represents the resultant of these two forces.

Exp. 18.—Verification of the Parallelogram of Forces.—

(*a*) MECHANICAL DETAILS.—Take three strings; knot them together in a point A (Fig. 25). To their ends attach any three weights P, Q, R, say P, Q, R lbs., respectively (any two of which are together greater than the third). Allow one string to hang freely with its suspended weight R, and pass the other two over two very **smooth and smoothly running pulleys** H, K, fixed in front of a vertical board.

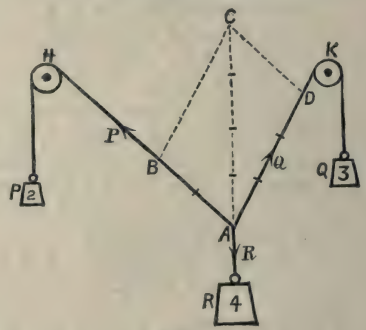


Fig. 25.

(*b*) GEOMETRICAL CONSTRUCTION. — When the strings have taken up

a position of equilibrium, note the position of A and the directions of the strings AH, AK by pencil marks on the board; remove the strings and measure off on the *board* lengths AB, AD, containing P and Q units of length along AH, AK, respectively. Complete the parallelogram $ABCD$, and join AC.

(*c*) OBSERVED FACTS.—*Then it will be invariably found*

(i.) *that the diagonal AC is vertical,*

(ii.) *that AC contains R units of length.*

(*d*) DEDUCTIONS.—Now the knot A is in equilibrium under the pulls P, Q, R acting along the strings, respectively. Therefore the force which may replace P, Q is equal and opposite to the weight R, that is, a force R, acting vertically upwards.

$\therefore$ AC represents the resultant in magnitude and direction.

Fig. 25 is drawn for the case in which $P = 2$ lbs., $Q = 3$ lbs., $R = 4$ lbs. The measured lengths AB, AD must therefore contain 2 and 3 units of length respectively. When the parallelogram is constructed, the diagonal AC, passing through the point of application of the three forces, will be found to be vertical and to contain 4 units of length.

34. Graphic Method of estimating the Resultant of two Forces acting on a Particle.—

Draw two straight lines AB, AD to represent the given forces in magnitude, line of action, and direction. For this a scale and protractor are required. Complete the parallelogram $ABCD$ and measure the length of the diagonal AC. This will give the magnitude of the resultant on the same scale as that on which AB and AD represented the given forces. The angle which its line of action makes with the line of action of either force can be found by means of the protractor.

35. *Example.*—To find graphically the resultant of forces of 7 lbs. and 11 lbs., whose directions include an angle of 60°.

Take any unit of length and measure off AB, AD (Fig. 26) containing 7 and 11 units respectively, making

$$\angle BAD = 60°.$$

Complete the parallelogram $ABCD$.

Then AC represents the resultant.

On AC mark off from A a scale of the selected units. Then C will be found to lie between the 15th and 16th marks, so that AC contains about $15\frac{3}{4}$ units.

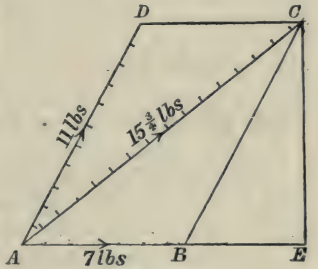


Fig. 26.

Therefore the resultant force = $15\frac{3}{4}$ lbs. wt. roughly.

Exp. 19.—A nail is driven into the wall, and a weight of 50 grammes is suspended from it. To a point on the cord between the nail and the weight another cord is fastened, and force is applied horizontally until the upper part of the former cord makes an angle of 30° with the vertical. Find, by a graphic construction, the tension in the horizontal cord, and verify the result experimentally. This should be done by an obvious modification of Fig. 25.

Note.—In making a graphic construction, the figure should be drawn to cover at least half a page, in order to make the percentage error small.

EXERCISES III.

PRACTICAL.

1. Draw a diagram, as well as you can to scale, showing the resultant of two forces, equal to the weights of 6 and 12 lbs., acting on a particle, with an angle of 60° between them; and, by measuring the resultant, find its numerical value.

2. Find graphically the magnitude of the resultants of the following pairs of forces acting at right angles. Find also the inclination of each resultant to the larger force:—

(i.) 40 lbs., 30 lbs. (iii.) 5 tons, 3 tons.

(ii.) 60 lbs., 25 lbs. (iv.) 10 tons, 9 tons.

3. What is meant by the resultant of two forces?

Draw a diagram to scale showing the resultant of two forces acting at a point, one of them being a force of 4 kilos. acting from north to south, and the other a force of 1 kilo. acting from south-east to north-west.

4. Two forces, the magnitudes of which are proportional to the numbers 3 and 4, act on a point at right angles to each other. Draw a parallelogram as nearly to scale as you can to show the direction and magnitude of the resultant, and deduce by measuring your diagram, or in any other way, the magnitude of the resultant and the angle it makes with the smaller force.

36. The Triangle of Forces.—

Suppose there are two forces acting on a body which can be represented in magnitude and direction by AB and AD (Fig. 27). Construct AC to represent the resultant by the parallelogram law.

There is, however, a somewhat easier construction for the resultant. This we proceed to give; the student should himself be able to see that it is justified.

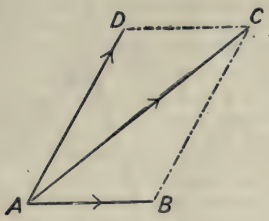


Fig. 27.

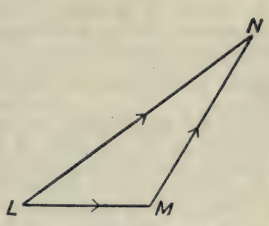


Fig. 28.

From any point *L* (Fig. 28) draw *LM* parallel and proportional to *AB*.

From the extremity *M* draw *MN* parallel and proportional to *AD*.

Join *LN*.

LN is then parallel and proportional to the resultant of the two given forces.

NOTE.—(i.) That *LN*, not *NL*, is the direction of the **resultant.**

Furthermore, the student should see that, if, in addition to the two forces represented by *LM* and *MN*, there were a third force, which could be represented in magnitude and direction by *NL*, acting at the point of intersection of the first two, there would be **equilibrium;** for the third force *NL* would be equal and opposite to the resultant *LN* of the first two.

NOTE.—Two sides of a triangle are together greater than the third; hence, for three forces, acting along different lines, to be in equilibrium, each force must be less than the sum of the other two. [See also Exp. 17.]

The Triangle of Forces.—If three forces (i.) act at one point in a body, and (ii.) are proportional to the sides of a triangle taken in order, the forces will be in equilibrium.

Conversely.—**If three forces acting at one point in a body are in equilibrium, they may be represented by the sides of a triangle taken in order.**

NOTE.—Suppose a pencil point to be placed at any vertex *A* of a triangle *ABC*, and suppose it to move thence round the triangle either way continuously, *i.e.* without retracing even the shortest portion of the path: it is then said to trace the sides of the triangle *in order*.

Examples.—(1) Forces of 12, 14, 15 units act away from a point and are in equilibrium. Show, by a diagram, how they act, and note the angles between their directions.

Draw $\triangle ABC$ (Fig. 29), whose sides are 12, 14, and 15 units. (Euc. I. 22.)

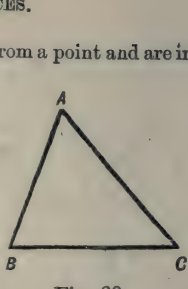


Fig. 29.

Then, by the Triangle of Forces, three forces in proportion to the sides of $\triangle ABC$, and acting parallel to those sides, are in equilibrium.

Also the relative directions of the given forces are represented by AB, BC, and CA; for all triangles of sides proportional to 12, 14, 15 are exactly similar, *i.e.* equiangular, to each other.

Take a point O (Fig. 30). Draw OP parallel and equal to AB, OQ parallel to and equal to BC, and OR parallel to and equal to CA; then Fig. 30 shows how the forces act.

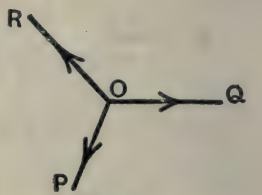


Fig. 30.

Measure the angles POQ, QOR, ROP with a protractor. They are 110°, $133\frac{1}{4}$°, $118\frac{3}{4}$° respectively. Note that these angles are the supplements of the angles ABC, BCA, CAB; so that perhaps in practice more accurate results will be got by measuring those angles and subtracting their values from 180° instead of measuring the angles around O.

(2) A small ball K of mass $\frac{1}{2}$ lb. is fastened to a string PK whose end P is attached to a wall PM. The ball is pulled away from the wall by means of a string held horizontally until PK makes an angle of 60° with the wall. Find the tensions of the strings.

Draw a figure (like Fig. 31) and letter it.

The forces acting on the ball are

(i.) its weight of $\frac{1}{2}$ lb. acting vertically down;

(ii.) the tension T of the string PK acting along KP;

(iii.) the tension F of the horizontal string acting horizontally.

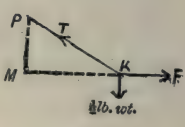


Fig. 31.

To the figure add arrows to denote these forces.

If, now, we draw KM perpendicular to the wall, $\triangle PMK$ is a triangle of forces; for

the tension T acts along KP in the direction from K to P;
the $\frac{1}{2}$ lb. wt. acts along PM in the direction from P to M;
the tension F acts along MK in the direction from M to K.

(Note that, relative to a tracing point going from K to P, P to M, and M to K, the arrows always point onward, and that therefore the sides are *taken in order*.)

Also the ball is in equilibrium;

Therefore, by the converse of the Triangle of Forces, the sides KP, PM, MK represent the forces in magnitude.

Now $\angle MPK$ is 60°, and $\angle PMK$ is 90°.

Therefore $\triangle PMK$ is a semi-equilateral triangle.

$$\therefore \quad KP = 2PM \quad \text{and} \quad KM = \sqrt{3}PM.$$

But PM represents the weight of the ball, *i.e.* $\frac{1}{2}$ lb. wt.;

$\therefore$ tension T, represented by KP, $= 2 \times \frac{1}{2}$; or 1 lb. wt.

and tension F, represented by MK, $= \sqrt{3} \times \frac{1}{2}$, or $\frac{1}{2}\sqrt{3}$ lbs. wt.

37. The Resolution of Forces. — Just as any pair of forces, acting at a point, can be replaced by a single force that will produce the same effect, so any single force can be replaced by two forces which, acting together at the same point, will produce the same effect as the single force. Two such forces will be those represented by the adjacent sides of any parallelogram whose diagonal, passing through the same point, represents the single force.

This process of finding a pair of forces equivalent to a single force is called the *Resolution of the Force.* Generally a force is resolved along two lines at right angles to each other.

Exp. 20.—To find the resolved parts of a force of 12 units along two axes at right angles, one axis making an angle of 30° with the line of action of the force, set off a line OR (Fig. 32), representing 12 units. At one end, O, draw a line, OB, making an angle of 30° with OR. Through O draw OA, at right angles to OB, on the opposite side of OR. Through R draw RP, RQ parallel to these lines. Then OP, OQ represent the components in magnitude and direction.

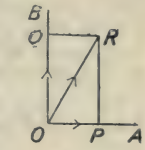


Fig. 32.

EXERCISES III. (*continued*).

5. State the condition of the equilibrium of three forces acting at a point, called the "triangle of forces." Find, in any way, how forces of 11, 9, and 3 units must act at a point if they are in equilibrium.

6. Forces of 10, 13, and 16 units act at a point and are in equilibrium. Find, by a diagram, how they act, and note the angles between their lines of action.

7. Forces of 5, 4, and 10 units act at a point. Can these be in equilibrium? If not, why not?

8. The line of action of one force makes angles of 90° and 120° with those of two other forces, and the forces are in equilibrium. Draw a diagram showing how the forces act, and compare their magnitudes.

Summary.—Chapter III.

1. The magnitude and direction of the resultant of two forces acting on a particle are obtained thus: (*a*) From any point draw two straight lines parallel to the forces; (*b*) cut off lengths proportional to these forces; (*c*) complete the parallelogram with these two lines as adjacent sides; (*d*) draw the diagonal through the original point.

This diagonal represents the resultant in magnitude and direction.

This is based upon the principle called the *Parallelogram of Forces.* (§§ 33–35.)

2. A second way of determining the magnitude and direction of the resultant of the two forces is as follows:—(*a*) From any point *A* draw a straight line *AB* parallel to one of the forces and in its direction. (*b*) From *B* draw a second straight line *BC* parallel to the second force and in its direction. (*c*) Make *AB* and *BC* proportional to the forces to which they are respectively parallel. (*d*) Join *AC*.

AC represents the resultant in magnitude and direction.

This is based on the principle called the *Triangle of Forces.* (§ 36.)

EXERCISES III. (*concluded*).

9. A force of 40 units acts in a North Easterly direction. Find its components in the North and East directions (1) graphically, (2) by calculation.

10. A force of 100 units acts in a direction 30° West of a line drawn South. Find its South and West components (1) graphically, (2) by calculation.

11. A force of 10 units acts vertically upwards. Find its horizontal component.

12. Draw a parallelogram *ABCD* and its diagonal *AC*. Take any point *O* in *AC* or *AC* produced. Show that the areas of the triangles *AOB*, *AOD* are equal. Let *AB*, *AD* represent two forces and *AC* their resultant. Show that the area of the $\triangle AOB$ is numerically equal to twice the moment of the force *AB* about *O*. Hence prove that the moments of two forces about any point in the line of action of their resultant are equal and opposite.

13. Describe a simple apparatus for testing the truth of the "parallelogram of forces" for two forces each equal to a weight of 30 grammes and producing a resultant equal to a weight of 50 grammes. Draw a careful

diagram showing how you would use the apparatus in the particular case given, representing the "parallelogram of forces" in its correct shape.

14. Illustrate by a drawing and an example the parallelogram of forces.

15. Forces of 7 and 16 units have a resultant of 21 units. Find the angles between the lines of action of the forces by a construction drawn to scale.

16. Two forces, each of 10 units, have a resultant of 5 units. Find, by a construction, the angle between the lines of action of the two forces.

17. Draw the triangle *ABC*, whose sides *BC*, *CA*, *AB* are 7, 9, 11 units long. If *ABC* is the triangle for three forces in equilibrium at a point *P*, and if the force corresponding to the side *BC* is a force of 21 lbs., show, in a diagram, how the forces act, and find the magnitude of the other two forces.

18. A mass of 24 lbs. is suspended by two flexible strings, one of which is horizontal and the other inclined at an angle of 30° to the vertical. What is the tension in each string?

19. State the "parallelogram of forces." You are provided with three small spring balances (sometimes called "dynamometers"), a blackboard, chalk, string, &c. How can you verify the proposition?

20. A man tows a boat up-stream by two cords, one attached to the bow and one to the stern. Of what advantage is this arrangement? Show by a diagram the forces acting on the boat.

21. A small ring is laid on the middle of a round table. Three strings, supporting weights of 13 lbs., 24 lbs., and 37 lbs. respectively are fastened to the ring, and the weights are allowed to hang over the edge of the table. It is required so to arrange the weights and strings that the ring may remain at rest. Explain the possible arrangement or arrangements (1) when the table is so smooth that friction may be neglected, (2) when friction has to be taken into account. Draw a diagram to illustrate your answer.

CHAPTER IV.

THE INCLINED PLANE.

38. It is sometimes more convenient to raise a weight gradually to a desired level by pulling or pushing it up a sloping surface than by lifting it vertically. Such a sloping surface may be called an **inclined plane.**

Examples.—Coal and ore are often brought to the mouth of a furnace by means of inclined trolley lines. Railway trains are often taken over hills several hundred feet high by means of inclines. Beer barrels are often raised from cellars or from the ground into brewers' drays by rolling them up two sloping wooden rails.

39. Mechanical Advantage.—The ratio of the weight of the body to the effort required to support the body on the slope is called the **mechanical advantage** of the inclined plane.

Denoting the effort required by P, and the weight of the body by W, we get

$$\textbf{mechanical advantage} = \frac{W}{P}.$$

Examples.—(1) If the weight of the body is 10 lbs. wt. and the effort required to support it is 2 lbs. wt.,

$$\text{the mechanical advantage} = \frac{10}{2} = 5.$$

(2) If the mechanical advantage is 3, what effort is required to support a mass of 21 gms. ?

$$\text{Mechanical advantage} = \frac{\text{weight of body}}{\text{effort}};$$

$$\therefore \quad 3 = \frac{21 \text{ gms. wt.}}{\text{effort}}, \quad \text{or effort} = \frac{21 \text{ gms. wt.}}{3} = 7 \text{ gms. wt.}$$

40. The Inclined Plane.—An inclined plane may be represented by a right-angled triangle *ABC* (Fig. 33), one side of which, *AC*, is horizontal, and is called the *base ;* a second side, *BC*, is vertical, and is called the *height ;* while the other side, *AB*, is called the *length*. These three sides are usually represented by the letters b, h, l respectively. The *inclination of the plane* to the horizontal is the angle *BAC*.

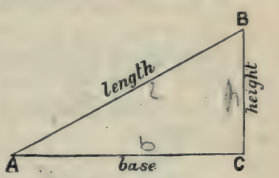


Fig. 33.

Frequently an inclined plane is spoken of as *rising* 1 *in* n. This means that for every n ft. (or yds. or ins.) a person walks straight up the plane he rises *one* ft. (or yd. or in.) vertically. Thus $1 : n$ expresses the ratio of *BC* to *AB*, or h to l.

Thus $h \div l = \frac{1}{100}$ when the plane rises 1 in 100.

An inclined plane is *smooth* when its surface offers no resistance parallel to its surface to a body rolling or sliding along it.

In this chapter it is always assumed that the inclined plane is smooth.

No inclined plane is absolutely smooth : there are always imperfections due to roughness of surface. Hence the actual mechanical advantage of an inclined plane is always less than that calculated theoretically in the following pages.

Exp. 21.—Obtain two pieces of smooth board, *AB*, *AC* (Fig. 34), about 24 ins. long and 4 ins. wide. Hinge them together at the end *A*. Clamp *AC* to the bench or table. If a small block of wood, *D*, be inserted between the boards, *AB* may be inclined at any desired angle. We thus obtain an inclined plane which is easily adjusted to any slope.

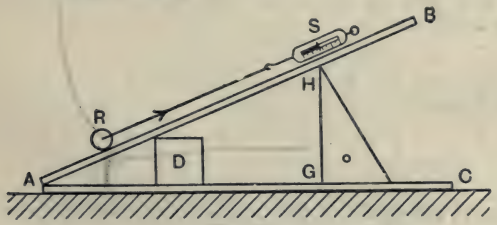


Fig. 34.

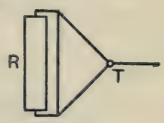


Fig. 34A.

Obtain a small, fairly heavy, smooth metal cylinder *R* mounted smoothly on a framework provided with a hook *T* (see Fig. 34A), to which can be attached a string. It is very convenient if the apparatus be so contrived that when the string is pulling *R* up the plane the string may lie either parallel to the plane or horizontal. A slot down the middle of *AB* will serve the latter purpose.

Test the cylinder and plane for friction by placing *R* on *AB* and slightly tilting *AB*. If the friction is small, as it should be, the cylinder will begin to move almost as soon as *AB* is tilted at all.

Weigh the cylinder and the attached framework and adjust the plane to any convenient slope. Tie a piece of cotton to *T* and to the spring balance, place the cylinder on the plane, and hold the spring balance so that the string lies parallel to the slope; the spring balance will register the force P which, acting parallel to the plane, is necessary to support the weight W of the cylinder and framework.

It will be found that, owing to friction, the force required to keep the body at rest will vary between certain limits. The less the friction the smaller this range will be. To get a definite value, find first the least force required to make the cylinder move upwards, then the least force that will just prevent it moving downwards. The mean of these two may be taken as the equilibrating force, *i.e.* $P = \frac{1}{2}(P_1 + P_2)$.

In order to measure the height, length, and base of the plane, place a large set square underneath as shown. *HG* gives accurately the height of the plane for a length *AH* and base *AG*; *AH* and *AG* can easily be measured by a millimetre scale and *HG* can be measured once for all.

In this experiment measure *AH*, *HG*. Repeat the experiment many times with different inclinations for *AB*. Tabulate the results thus :—

GH	*AH*	P	W	*GH*/*AH*	P/W

Observation.—It will be found that the corresponding values in the fifth and sixth columns are equal.

Deduction.—When the supporting force is *parallel to the plane* the following law holds :—

$$\frac{\text{supporting force}}{\text{weight}} = \frac{P}{W} = \frac{GH}{AH} = \frac{\text{height of plane}}{\text{length of plane}}.$$

Exp. 22.—Repeat the last experiment, keeping the string *horizontal.* To do this a long central slot must be cut out of the middle of the sloping board *AB* of the last experiment. The string must lie horizontally through this slot. Measure *GH, AG* and tabulate the results thus :—

AG	*GH*	*W*	*P*	*GH/AG*	*P/W*

Observation.—The value in the fifth column is always equal to the corresponding value in the sixth.

Deduction.—When the supporting force is *horizontal,*

$$\frac{\text{supporting force}}{\text{weight}} = \frac{P}{W} = \frac{GH}{AG} = \frac{\text{height of plane}}{\text{base of plane}}.$$

41. Application of the Triangle of Forces to the Inclined Plane.—When a body is supported on a smooth inclined plane the forces acting on the body are

(i.) its weight W;

(ii.) the supporting force, P;

(iii.) the force, R, exerted by the plane and termed the *reaction* of the plane.

Since the plane is supposed smooth, it does not exert any resistance parallel to its length, and therefore the only force it exerts is *perpendicular* to its length.

When the direction of the supporting force is known the relation between these three forces W, P, and R can be found by the Triangle of Forces.

(i.) *Equilibrium of a body on a smooth inclined plane when the supporting force acts* **horizontally.**

Let O (Fig. 35) be the position of a body kept at rest on a smooth inclined plane ABC by means of a *horizontal* force P. Let W be the weight of the body, and R the reaction.

It is required to find the relations between P, W, and R.

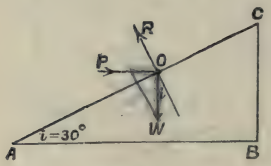


Fig. 35.

Fig. 36.

The three forces acting on the body at O are

(i.) the weight W acting vertically, *i.e.* perpendicular to AB;

(ii.) the force P acting horizontally, *i.e.* perpendicular to BC;

(iii.) the reaction R acting perpendicular to the plane at the point O, *i.e.* perpendicular to CA.

Therefore the three forces W, P, R act perpendicular to AB, BC, CA respectively.

Draw the Triangle of Forces $A'B'C'$ (Fig. 36.)

Then

$P : W : R ::$ length of $B'C'$: length of $A'B'$: length of $C'A'$;

i.e.

$$\frac{P}{B'C'} = \frac{W}{A'B'} = \frac{R}{C'A'} \quad \ldots\ldots\ldots\ldots \quad (1)$$

Also it is evident that the triangles ABC, $A'B'C'$ are similar triangles, since the sides of one triangle are respectively perpendicular to the sides of the other: therefore

$$\frac{BC}{B'C'} = \frac{AB}{A'B'} = \frac{CA}{C'A'} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2).$$

$\therefore$ from (1) and (2),

$$\frac{P}{BC} = \frac{W}{AB} = \frac{R}{CA},$$

or

$$\frac{P}{h} = \frac{W}{b} = \frac{R}{l},$$

where b, h, l are the base, height, and length respectively of the inclined plane.

$$\therefore \quad P = \frac{h}{b} W = \frac{\text{height of plane}}{\text{base of plane}} W,$$

and

$$R = \frac{l}{b} W = \frac{\text{length of plane}}{\text{base of plane}} W.$$

(ii.) *Equilibrium of a body on a smooth inclined plane when the supporting force acts* **along the plane.**

Let O (Fig. 37) be the position of a body kept at rest on a smooth inclined plane by means of a force P acting along the plane. Let W be the weight of the body, R the reaction of the plane.

It is required to find the relations between P, W, R.

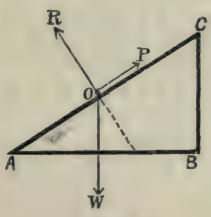


Fig. 37.

Fig. 38.

The three forces acting on the body at O are

(i.) the weight W acting vertically;

(ii.) the force P acting along the plane;

(iii.) the reaction R acting perpendicular to the length of the plane.

Draw the Triangle of Forces DEG (Fig. 38). Then

$$\frac{P}{EG} = \frac{W}{GD} = \frac{R}{DE} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1).$$

But it is evident that the triangles ABC, DEG are equiangular and therefore similar (viz. $\angle G = \angle C$, $\angle E = \angle B$, $\angle D = \angle A$): hence

$$\frac{BC}{EG} = \frac{AC}{GD} = \frac{AB}{DE} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2).$$

$\therefore$ from (1) and (2),

$$\frac{P}{BC} = \frac{W}{AC} = \frac{R}{AB},$$

or

$$\frac{P}{h} = \frac{W}{l} = \frac{R}{b};$$

$$\therefore \quad P = \frac{h}{l} W = \frac{\text{height of plane}}{\text{length of plane}} W$$

and

$$R = \frac{b}{l} W = \frac{\text{base of plane}}{\text{length of plane}} W.$$

Example.—Find the force necessary to draw a smooth weight of 100 lbs. up an inclined plane whose inclination to the horizontal is 30° (i.) when the force acts horizontally, (ii.) when the force acts parallel to the plane.

When the angle of the plane is 30° $h : b : l = 1 : \sqrt{3} : 2$, as is evident from a figure.

(i.) $P = \frac{h}{b} W = \frac{1}{\sqrt{3}} 100 = \frac{100\sqrt{3}}{3} = 57{\cdot}7$ lbs.-wt.

(ii.) $P = \frac{h}{l} W = \frac{1}{2} 100 = 50$ lbs.-wt.

Summary.—Chapter IV.

1. The *mechanical advantage* of an inclined plane is the ratio of the weight of the body to the effort required to support it on the slope. (§ 39.)

2. In the *inclined plane* the relation between the effort and the resistance may be obtained by the triangle of forces. The relation is:—

(i.) with the effort horizontal $\frac{P}{W} = \frac{\text{height of plane}}{\text{base of plane}}$;

or (ii.) ,, ,, ,, up the plane $\frac{P}{W} = \frac{\text{height of plane}}{\text{length of plane}}$. (§ 41.)

EXERCISES IV.

1. A weight of 500 grammes is supported on an inclined plane by an effort parallel to the plane. The plane makes an angle of 30° with the horizontal. Find by a graphic construction the effort and also the reaction of the plane.

2. An inclined plane rises 3 ft. in 5 ft. What is its mechanical advantage when the force is (i.) parallel to the plane, (ii.) horizontal?

3. Explain the advantages of a zig-zag road up a hill. Find the steepest incline up which a force equal to the weight of 5 cwt. can move a weight of 2 tons.

4. A beer barrel with contents weighs 390 lbs. One end of a rope is attached to the top of an incline. The rope passes down the incline up around the barrel and back to the hands of a man who stands at the top of the incline. The incline being 1 in 3, find the least force which the man must exert in order to pull the barrel up.

5. A horse draws a trap weighing $5\frac{1}{2}$ cwt. up a hill rising 3 in 11. What is the pull in each trace?

6. Is it more advantageous in an inclined plane to have the force acting along the plane or horizontally?

7. What is meant by the *mechanical advantage* of a machine? Describe any simple device by means of which you could just support a weight of 5 lbs., using only a 1-lb. wt.

8. A railway incline slopes upwards at a slope of 1 in 100. An engine is drawing a train up this incline. The combined mass of the train equals 400 tons. Neglecting friction, calculate what must be the pull of the engine to maintain a slow and steady velocity up the incline.

CHAPTER V.

TIME. THE SIMPLE PENDULUM.

42. The Unit of Time.—We cannot separate the ideas of time and movement. Time is measured by movements that are known to be regular and uniform. From earliest days the apparent movement of the Sun round the Earth has been used for measuring time; for, although day and night vary in length at different seasons and in different places, it was thought that the interval taken by the Sun in moving from its highest altitude on one day to its highest altitude on the next day was always the same. Later it was found, by observation of the stars, that this interval, called the *solar day*, was not quite constant throughout the year. Therefore the average interval, called the *mean solar day*, is taken as the standard for comparison. This is divided into 24 equal parts, called hours, each hour into 60 equal parts, called minutes, and each minute into 60 equal parts, called seconds. Thus the second is the $\frac{1}{86400}$ part of a mean solar day.

43. Methods of Measuring Time.—To be able to divide the day into equal intervals we must observe some change that goes on regularly during the day, and that can be measured. It took many hundreds of years to discover such a change. The movement of a shadow cast by the Sun was tried as a time-measurer, it being assumed that the shadow moved through equal areas in equal times. But this sun-dial could be used only while the Sun was shining. To be able to measure time at any point of the day or night the flow of liquids was used. Water-clocks were devised in which water flowed continually from a reservoir, and the duration of time was taken as proportional to the amount of water that flowed out during that interval.

But it is known that the rate at which water flows from

a vessel varies with the pressure at the point of flow, and as this pressure is gradually diminishing in a water-clock, the rate of flow is not uniform. The burning of candles was another method utilised to compare intervals of time, but this again gave only approximate results. A sand-glass measures an interval of time that is of constant length. It does not, however, tell us what ratio this interval bears to the solar day.

44. The Pendulum.—Not until the beginning of the 17th century was a trustworthy means of obtaining uniform movement discovered. Then Galileo found that a well made pendulum vibrates at a constant rate, *i.e.* it makes an equal number of swings in equal times. For such a pendulum the time of swing is constant, and from the total number of swings made in a mean solar day the time of a single swing can be calculated.

For different pendulums the time of a single swing depends on the length of the pendulum, and it is possible to construct one that makes exactly 86,400 single swings in a mean solar day. Such an instrument is called a *seconds pendulum* because its time of swing is exactly one second. In a clock the vibration of a pendulum is used to maintain constant the rate of fall of a weight, and the motion is communicated, through cog-wheels and levers, to a finger moving over a dial. The finger moves through equal angles in equal intervals of time, and the amount of movement is registered on the dial.

Later it was found that the rate of uncoiling of a spring could be kept constant by means of a pendulum or by a vibrating wheel. In a watch the movement of the spring is maintained uniform by a wheel which is made to vibrate by being attached to a spiral spring called a "hair-spring."

45. The Simple Pendulum.—DEFINITION.—**A simple pendulum** *consists of a* **heavy particle** *attached by a* **weightless inextensible flexible string** *to a fixed point.*

There is no absolutely simple pendulum in Nature, for we cannot attach a string to a single particle, nor obtain a weightless or perfectly inextensible flexible string, *but a heavy piece of metal called the "bob" suspended by a long fine string, may be regarded as constituting approximately such a pendulum.*

Let P (Fig. 39) be the particle, A its position when at rest hanging by the string SA. In this position SA is vertical. If the particle is raised to C, the string being kept tight, and is then released, it will swing through the arc CA up to a point D on the other side of A at the same height above A as C is. It will then swing back again through A to C, and would continue to *oscillate* between C and D for ever were its motion not checked by the friction of the air and other resistances. The motion is termed **vibration** or **oscillation;** the distance AC or AD is called the **amplitude** of the vibration; and the time the pendulum takes to swing to *and* fro, *i.e.* from C to D and back again to C, is called the **time of a complete vibration,** or the **period** of the pendulum. The time of going to *or* fro, *i.e.* from C to D or from D to C, is called the time of semi-vibration or the time of a single swing (see § 44).

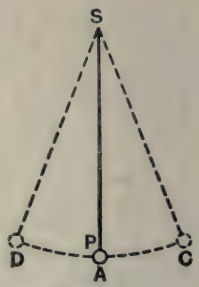


Fig. 39.

Exp. 23.—Set up a simple pendulum. Take a leaden ball or bullet and suspend it by means of a piece of silk or cotton thread about 1 metre long from a support which is best formed of two flat pieces of metal or wood clamped together and mounted in a retort stand as in Fig. 40. Place the retort stand near the edge of the bench so that the pendulum may hang over the side. When the pendulum is at rest drive a big pin into the edge of the bench or make a chalk mark on the edge to serve as a datum mark.

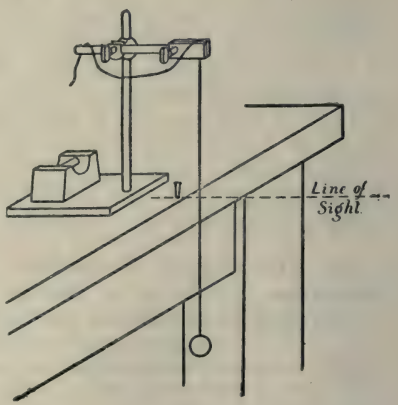


Fig. 40.

During the observations of the times of vibrations sit or stand opposite this mark.

Exp. 24.—To measure the period of vibration of a pendulum. Set the pendulum swinging. Stand 4 or 5 yds. off and note by a watch, or start a stop-watch at, the moment when the bob makes a *transit*, that is, passes in front of the mark. Again observe the time of transit when the pendulum has performed, say, 50 complete vibrations. This can be done by counting either each transit from right to left *or* the reverse way. Divide the interval by 50; the result is the period. Check this by timing another 50 swings.

We will now endeavour to find out how the time of vibration depends on (1) the amplitude of swing, (2) the mass of the bob, (3) the material of the bob, (4) the length of the thread.

Exp. 25.—*To find the relation between the amplitude and the period.* Keeping the length of string the same, repeat Exp. 24, several times, making the pendulum swing through arcs of different lengths (in no case must the angle of swing exceed 20°). The period will be found to be constant, thus showing that when the amplitude is small the period is independent of the amplitude, *i.e.* the vibrations are **isochronous.**

Exp. 26.—*To find the relation between the mass of the bob and the period.* Keeping the length of the string the same, repeat Exp. 24 with leaden bob of different size and mass. The period is the same, thus showing that the period is independent of the mass of the bob.

Exp. 27.—*To find the effect of a change in the material of the bob upon th period.* Keeping the length of the string the same, repeat Exp. 24 with bobs of cork, wood, stone, iron, &c. In all cases the period is the same, showing that the period of vibration does not depend on the kind of matter of which the bob is composed.

Exp. 28.—*To find the relation between the length of the pendulum and the period.* The length of the pendulum should be measured from the centre of the bob (if spherical) to the bottom of the cheeks which support the pendulum. Determine the period for, say, eight lengths varying between 10 cms. and 130 cms. Enter the lengths and corresponding periods in a table as shown below, which repre-

sents some figures actually obtained. In the third column enter the value of the quotient of the length of pendulum divided by the square of the period of vibration.

Length of Pendulum, in centimetres. l	Period in secs. t	$\frac{l}{t^2}$
129·5	2·28	24·9
114	2·15	24·7
99·4	2·00	24·9
80·3	1·79	25·0
65·7	1·62	25·0
50·1	1·43	24·5
37·5	1·24	24·4
24·3	·99	24·8
16·6	·82	24·7

Mean value of l/t^2 .. ... 24·8

Observe the constancy of $\frac{l}{t^2}$. From this we deduce experimentally that t^2 is proportional to l, *i.e.* **the period of vibration varies as the square root of the length.**

Instead of calculating l/t^2 we may calculate t^2 and plot a curve between l and t^2. The curve is a straight line showing that l/t^2 is constant. From the inclination of this line we can deduce the average value of l/t^2.

46. A mathematical treatment of the problem of the swing of the pendulum shows that the period (t) depends only upon the length (l) of the pendulum and the acceleration of gravity (g), the formula being

$$t = 2\pi\sqrt{\frac{l}{g}}.$$

At present you may not understand exactly what g means. If a body falls from rest solely under the attraction of the Earth, then after one second its velocity is 32·2 ft. per sec., after two seconds its velocity is 64·4 ft. per sec., after n seconds its velocity is $32{\cdot}2 \times n$ ft. per sec. The rate of increase of velocity, *i.e.* the acceleration, is therefore 32·2 ft. per sec. per sec., *i.e.* in the English system $g = 32{\cdot}2$ ft. per sec. per sec. In the metric system $g = 981$ cms. per sec. per sec. For further details, see Chap. VIII.

47. The Seconds Pendulum.—The "seconds" pendulum is a pendulum whose period is two seconds, *i.e.* its forward swing occupies 1 sec. and its backward swing occupies 1 sec.

Taking the formula $t = 2\pi\sqrt{\frac{l}{g}}$, squaring each side and transposing, we get $g = 4\pi^2\frac{l}{t^2}$.

From this, if l and t be known, g can be found: conversely, if l and g be known, t can be found.

Examples.—

1. From the mean value of l/t^2 given on page 54, find the value of g.

$$g = 4\pi^2 . \frac{l}{t^2} = 4 \times 9{\cdot}87 \times 24{\cdot}8 = 978 \text{ cms. per sec. per sec.}$$

2. Assuming the value of g as 981 cms. per sec. per sec., calculate the length of the seconds pendulum.

$$l = \frac{1}{4\pi^2} . gt^2 = \frac{1}{4 \times 9{\cdot}87} \times 981 \times 4 = 99{\cdot}3 \text{ cms.}$$

Summary.—Chapter V.

The unit of time is the mean solar day. It is equal to 86400 seconds. (§ 42.)

A pendulum and a vibrating spring afford the best means of measuring time. (§ 44.)

The period of a simple pendulum is independent of the angular amplitude (if this is less than 10°) and independent of the mass and material of the bob. It depends only upon the length of the pendulum and the acceleration of gravity. (§§ 45, 46.)

The formula for the period of a pendulum is

$$t = 2\pi\sqrt{\frac{l}{g}}. \qquad (\S\ 46.)$$

In the English system $g = 32{\cdot}2$ ft. per sec. per sec.;
in the metric system $g = 981$ cms. per sec. per sec. (§ 46.)

The seconds pendulum has a period of two seconds. The length of a seconds pendulum in England = 99·3 cms. (§ 47.)

EXERCISES V.

1. Two pendulums have exactly the same length. In one the bob is made of brass and swings through an arc of 5°. In the other the bob is made of iron and swings through an arc of 8°. Compare the times taken by each in making 20 swings.

2. The lengths of two pendulums are 16 cms. and 25 cms. Compare the times taken by each in making 50 swings.

3. A pendulum is made to beat seconds at London. When taken to Cairo it no longer beats seconds. Why is this?

4. Calculate the length in centimetres of a pendulum that will make one complete swing in 5 seconds.

5. A pendulum makes 36 complete swings in two minutes. Calculate its length in feet. (Take $\pi^2 = 10$ and $g = 32$).

6. One pendulum, A, makes 60 complete swings in two minutes. Another, B, makes 50 in $2\frac{1}{2}$ minutes. Compare the lengths of A and B.

7. What is the difference between a simple pendulum and the pendulum of an ordinary clock?

8. How would you show that the bob of a pendulum moves fastest when it is passing through its lowest position?

9. What is the function of the pendulum of an ordinary clock?

*CHAPTER VI.

COUPLES.

48. DEFINITION.—A **couple** consists of two equal forces acting in opposite directions along two parallel straight lines (see Fig. 41). *A couple cannot by itself keep a body in equilibrium*, for it tends to rotate the body—the points of application of the two forces tending to move in opposite directions. Moreover, the proof that two parallel forces have a single resultant fails for the case of a couple (§ 17).

Fig. 41.

Examples of couples.—In winding a clock we apply a *couple* to the key, for we do not try to make it move to one side or the other, but simply turn it round. To spin a small top between the finger and thumb we apply a *couple* to it by moving the finger and thumb sharply in opposite directions. To unlatch a door we apply a *couple* to the handle.

49. DEFINITIONS.—The **arm** of a couple is the perpendicular distance (*AB*, Figs. 42 and 43) between the lines of action of its two components (*i.e.* the two forces forming the couple).

The **moment** of a couple is the algebraical sum of the moments of its two components about any point in their plane. The two moments must be added if they tend to produce motions in the same direction, and subtracted if they tend to produce motions in opposite directions.

The following is the fundamental property of couples:—

* Chapters and articles marked with an asterisk are *not* required by the syllabus of the Preliminary Certificate Examination.

50. The moment of a couple is the same about all points in its plane.—Let the couple consist of two equal forces of magnitude P, one acting in a direction AC and the other in the opposite direction BD. Let O be any point in their plane. Draw OAB perpendicular to the forces.

Case I.—O does not lie between the two forces (Fig. 42). The moment about O of the force P acting along AC

$$= P \times OA.$$

The moment about O of the force P acting along BD

$$= P.OB.$$

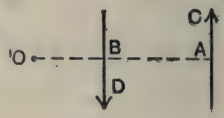


Fig. 42.

Since, considering OBA as a lever hinged at O, the forces tend to give opposite motions about O, the algebraical sum of the moments, *i.e.* the moment of the couple

$$= P.OA - P.OB = P(OA - OB) = P.AB.$$

Case II.—O lies between the two forces (Fig. 43). If we consider AB as a lever pivoted at O, it is evident that the two forces, P along AC and P along BD, both tend to rotate AB the same way; hence the moment of the couple

$$= P.OA + P.OB = P(OA + OB)$$
$$= P.AB,$$

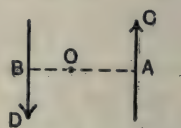


Fig. 43.

the same result as before.

Hence the moment of the couple about O is independent of the position of O and is equal to the product $P.AB$.

51. Alternative Expressions for the Moment of a Couple.—The **moment** of a couple may, therefore, be defined as—

(1) *The product of the measure of either force into the arm of the couple.*

(2) *The moment of either of the two forces about any point in the line of action of the other force.*

I.e. moment of couple $= P \times AB$

= moment about A of P acting along BD

= moment about B of P acting along AC.

52. A couple cannot be replaced by a single force.—For the moment of a single force about any point on its line of action is zero (§ 12). But the moment of a couple about every point in its plane is a constant quantity, differing from zero.

Hence a couple cannot have a resultant.

53. Two couples in the same plane whose moments are equal and opposite will balance each other.—This is obvious from the definition of moments. It may, however, be proved by the parallelogram of forces combining the forces in pairs or by experiment. (See below, Exp. 29.) From this theorem it follows that a couple may be replaced by any other couple of equal moment. From this again it follows that a couple has no particular place of application, but that it may be shifted anywhere in its plane without altering its statical effect. The effect of the couple depends, therefore, only on its moment and the plane in which it acts.

Exp. 29.—*To show that, if two couples acting on a body in the same plane balance, their movements are equal and opposite.*

Work on a bench. Apparatus required:—4 clamps, 4 spring balances, 4 nails, string, a piece of wood, some peas or marbles.

Drive 4 nails *a*, *b*, *c*, *d* (Fig. 44) into the piece of wood near the corners. Tie string to the nails. Place some peas or marbles on the bench and rest the wood on them. The peas or marbles give the wood an easy freedom of motion over the bench.

Take the clamps and fasten one to each side of the bench. Tie a spring balance to each of them. Take the string from each nail and loop it over the hook of the nearest balance. Arrange so that all the four balances are in tension and

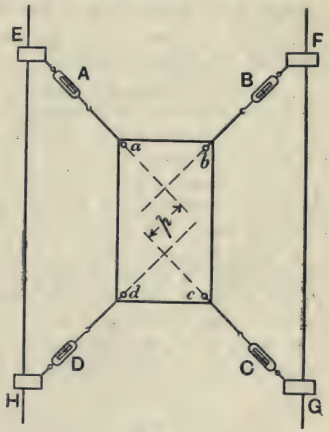


Fig. 44.

then adjust the clamps and lengths of the strings until the readings of A and C are the same and Aa and Cc are parallel. It will then be found that the readings of B and D are the same and that Bb and Dd are parallel.

Measure the distances p between Aa and Cc and q between Bb and Dd. Show now that

the reading of A (or C) $\times p$ = the reading of B (or D) $\times q$,

i.e. the moments of the two couples under which the wood is in equilibrium are equal and opposite. Repeat the experiment with the clamps in different positions.

Summary.—Chapter VI.

A couple consists of two equal forces acting in opposite directions along two parallel straight lines. (§ 48.)

The moment of a couple equals the product of either force into the arm of the couple. (§§ 50, 51.)

A couple cannot be replaced by a single force, but it can be replaced by a couple of the same moment in the same plane. (§§ 52, 53.)

EXERCISES VI.

1. What is meant by a *couple* in mechanics? Show that the moment of a couple is the same about all points in its plane.

2. State (without proof) the conditions that must be satisfied in order that two couples may balance.

3. Show that a couple has no particular point of application, but may be shifted anywhere in the same plane without disturbing the equilibrium of the body to which it is applied. Is this true of a force? Explain the difference, if any.

4. The forces of a certain couple are each 16 dynes, and the arm of the couple is 10 cms. A couple which balances this has an arm of 8 cms. Find the magnitude of the forces of this couple.

5. Three forces act along the sides of a triangle taken in order and are proportional to the sides along which they act, their magnitudes being $P.AB$, $P.BC$, $P.CA$. Find the moment of the couple which is equivalent to this system of forces.

6. Show that a force acting on any point in a body is equivalent to an equal and parallel force acting at any other point and a couple.

*CHAPTER VII.

VELOCITY.

54. Speed and Velocity. — In § 3 *motion* was defined as *change of position*. It is not, however, usually sufficient to know that a body is in motion. We may wish to learn whether a body is moving quickly or slowly, and to be able to compare its motion with the motions of other bodies. For this purpose, we make use of the terms *speed* and *velocity*.

DEFINITIONS.—*The* **speed** *of a body is its* **rate of change of position** *when the* **line along which it is moving** *and the* **direction** *along that line are* **not taken into account.**

The **velocity** *of a body is its* **rate of change of position** *when the* **line along which it is moving** *and the* **direction** *along that line are* **taken into account.**

To specify a speed completely, it is necessary to state only its *magnitude;* whereas

To specify a velocity completely, we must state not only (i.) its *magnitude*, but also (ii.) the *line of motion* of the body, and (iii.) the *direction* along that line.

NOTE.—In cases where the line of motion is evident, and there is no ambiguity, we use the term *velocity* when referring only to the *magnitude* of the velocity and the *direction* along the line of motion.

The following illustration will help to make the difference between velocity and speed clear.

Suppose two men, *A* and *B*, to be walking in opposite directions along the road from Manchester to Liverpool. If *A* and *B* each traverse a mile in every quarter of an hour, the *speed* of each can be said to be 4 miles per hour, but, if *A*'s *velocity* is 4 miles per hour, *B*'s cannot be the same, for he is moving *in the opposite direction*. To distinguish between these velocities, we make use of the terms *positive* and *negative*. (See § 56.)

* See footnote on page 57.

55. Representation of Velocities by Straight Lines.—The motion of a body at any instant is therefore fully determined when

(i.) The *line of motion*, *i.e.*, the line along which the body is moving,

(ii.) the *direction* along that line,

(iii.) the *speed*, or magnitude of the velocity, with which it is moving

are known.

Now these three elements can be represented by an arrow-headed line. The line should be drawn parallel to the line of motion, the arrow-head drawn to indicate the direction along the line, and the length of the line should contain as many units of length as the velocity contains units of velocity.

It is very important to notice that for this unit of length we need not choose a foot or a centimetre. What length is chosen is perfectly immaterial. It can be *any convenient distance*, but, having fixed upon it for the representation of that particular velocity, we must use it for *all* velocities in the same problem.

We thus see that by a velocity represented by CD, where CD is a straight line, we mean a velocity having the following elements :—

line of motion, CD; direction from C to D;

speed, equal to the units of length contained by CD on some scale specified or understood.

It is important to notice that the direction of the motion is indicated by the order in which the letters C, D are mentioned. Thus, if a body has a velocity represented by DC, it is moving along the line DC in the direction *from* D *to* C and with a speed of as many units of speed as DC contains units of length.

The relative directions of lines of motion can be specified by stating what angles they make with certain fixed straight lines: *e.g.* the vertical and the "points of the compass."

56. Positive and Negative Velocities.—If the velocity of a body moving in a certain direction along a straight line be considered *positive*, then the velocity of any body moving in the opposite direction along the same or a parallel line is considered *negative*.

NOTE.—*The terms positive and negative are applied also to any direction, and the opposite direction, as well as to the velocities in those directions.*

In the example given in § 54, if *A*'s velocity be taken as positive, *i.e.* +4 miles per hour, then *B*'s velocity is negative, and equal to −4 miles per hour.

Caution.—It is usually unimportant which direction be chosen as positive, but care must be taken in any particular example that, after the selection of the positive direction, *all* velocities in that direction be considered positive, and *all* velocities in the opposite direction negative.

57. Uniform and Variable Velocities. — DEFINITIONS. —*The velocity of a body is said to be* **uniform** *when the body traverses equal distances in equal intervals of time*, **however small.**

When this is not the case, the velocity of the body is said to be **variable.**

In order to ensure a full appreciation of the definitions, and of the importance of the words *however small*, which must be carefully noted, we will take a simple illustration.

A train from London to Scotland passes four stations, *A*, *B*, *C*, *D*, on the route, 30 miles away from one another, at 1 o'clock, 2 o'clock, 3 o'clock, and 4 o'clock respectively. It therefore traverses *equal distances* (30 miles) in *equal intervals of time* (1 hour). It *may* have been moving at the same rate throughout, but this is not at all likely. It is more probable that the train stopped at some intermediate stations. The fact that the train traversed 30 miles in each of the 3 hours does not warrant us in thinking that it was moving at the same rate for the whole of the time.

Suppose, now, a passenger notes by his watch that the train takes just 2 mins. from milestone to milestone for a distance of 10 miles. Here, again, equal distances (1 mile) are traversed in equal intervals of time (2 mins.). In this case it seems probable, but still by no means certain, that the train was moving uniformly over the 10 miles.

If the passenger found further that the times between consecutive telegraph poles, which he knows to be 66 yds. apart, were, for the distance of a mile, all equal to $4\frac{1}{2}$ secs., he would be *almost* certain that the train had been moving uniformly over the observed mile.

We thus see that it is only by noting the distances passed over in *very small* intervals of time that we can ascertain whether the velocity of a body is uniform or not. It is not sufficient to know that equal distances are passed over in equal times; but, if equal distances are described in equal intervals, *however small*, the velocity is uniform; if not, the velocity is variable.

58. Measure of Uniform Velocity. — Velocity, *when uniform, is* measured by *the* distance traversed in a unit of time.

The **unit of velocity** is the velocity of a body that moves uniformly over unit distance in a unit of time.

Therefore the **F.P.S. unit of velocity** is a velocity of **1 foot per second** and the **C.G.S. unit of velocity** is a velocity of **1 centimetre per second.**

The reader is specially warned against speaking of a "velocity of 40 feet," say. **If the unit of distance is mentioned, the unit of time must also be mentioned.** One must therefore speak of a velocity of 40 *feet per second*," or, merely, of "a velocity 40," in which case the units of distance and time are understood. In the C.G.S. system "a velocity 40" would mean a velocity of 40 *centimetres per second*.

In the case of the velocities of ships, it is right to say "a velocity of 20 knots," for a knot is the nautical unit of velocity and is equal to 1 nautical mile per hour (1 nautical mile = 6077 feet).

59. To find the distance in feet passed over in t seconds by a body moving uniformly with a velocity of u feet per second.

Let s be the number of feet traversed in t secs.

By definition (§ 58), the body moves over u ft. in *each* second.

Therefore in 1 sec. the distance traversed is u ft.;

" 2 " " " " $2u$ " ;

" 3 " " " " $3u$ " ;

and generally in t " " " " tu " ;

$$\therefore \quad s = tu;$$

or, as it is more usually written,

$$s = ut \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1).$$

60. In the preceding section a foot and a second were chosen as the units of time and distance. But it is important to notice that formula (1) and others that we shall obtain are **true whatever units we select, provided we keep to the same units throughout** the investigation.

Thus, if t be expressed in *seconds* and u in *centimetres* per *second*, then s is expressed in *centimetres;* or, if t be expressed in *hours* and u in *miles* per *hour*, s is expressed in miles.

By bearing this in mind, the reader will be able frequently to curtail the work necessary in solving problems.

Examples.—(1) A body is moving at the rate of 80 ft. per sec. How far does it go in 7 mins.?

Here $u = 80\ ft.$ per $sec.$ and $t = 7\ mins. = 420\ secs.$

$$\therefore\ s = ut = 80 \times 420 = 33{,}600\ ft.$$

(2) A body is moving at the rate of 60 miles an hour. Express this velocity in feet per second.

60 miles $= (60 \times 5280)$ ft., and 1 hour $= (60 \times 60)$ secs.

Therefore the body is moving at the rate of

(60×5280) ft. in (60×60) secs., *i.e.* $\dfrac{60 \times 5280}{06 \times 60}$ or 88 ft. per sec.

Example (2) is a case of what is termed **change of units,** and the method followed should be mastered. The result there obtained is important, and its use, whenever possible, will effect a saving of time, and diminish the liability to mistakes. Remember that

60 miles per hour = 88 ft. per sec. ..... (2).

Example.—10 miles an hour $= \frac{1}{60}$ of 60 miles per hour $= \frac{1}{60}$ of 88 ft. per sec. $= 14\frac{2}{3}$ ft. per sec.

61. Average or Mean Velocity.—

DEFINITION.—**The average or mean velocity** *of a body* A *during a stated interval is the velocity of another body* B *which,* **moving uniformly,** *would pass over an* **equal distance in the same time.**

Let t be the time taken by A to describe the distance s.

Then B describes the distance s in time t *uniformly,* and therefore, by formula (§ 59), the distance B travels is $\dfrac{s}{t}$.

Therefore the **average velocity** of A is $\frac{s}{t}$, or the

distance traversed = average velocity × time.

Example.—If a train travels from London to Plymouth, a distance of 224 miles, in 4 hours, making various stoppages on the way, its average velocity equals $\frac{s}{t} = \frac{224}{4} = 56$ miles per hour. Any other train that moved at the rate of 56 miles per hour through the whole distance would take the same time to do the journey.

62. Relative Velocity.—The rate at which a body A approaches or recedes from another body B travelling along a parallel or in the same straight line with A is called the velocity of A relative to B.

Examples.—(1) A train of length 250 ft. travelling at 40 miles an hour overtakes a train of length 300 ft. travelling at 15 miles an hour. How long will the first train take to pass the second?

The relative velocity of the first train to the second (*i.e.* the rate at which the first gains on the second) is $(40-15)$ or 25 miles an hour. Thus the time required will be the same as if the second train were at rest and the first were travelling at 25 miles an hour.

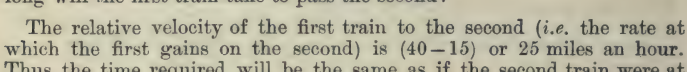


Fig. 45.

Assuming the second train at rest in the position CD (Fig. 45), the first train (while passing it) moves from the position AB to the position EF. Thus the distance travelled by the first train while passing the second is BF or 550 ft.

Thus we calculate how long it will take a train travelling at 25 miles an hour to move through 550 ft. Since 25 miles an hour $= \frac{110}{3}$ ft. per sec., the answer is $550 \div \frac{110}{3}$ or 15 secs.

(2) How long will the same trains take in passing if they are travelling in opposite directions?

Their relative velocity is now $(40+15)$ or 55 miles an hour. Hence we may suppose the second train to remain at rest in the position CD, while the first travels from the position AB to the position FF at the rate of 55 miles an hour, or $\frac{242}{3}$ ft. per sec.

Hence the required time is $550 \div \frac{242}{3}$ or $6\frac{6}{11}$ secs.

63. Component and Resultant Velocities. — DEFINITIONS.—The actual velocity of a body A may be due to its velocity relative to a body B on which it is moving and the velocity of B itself. In such a case the two latter velocities are called **component velocities,** and the actual velocity of A is called the **resultant velocity.**

Example.—A man rows along a river flowing at the rate of 1 mile per hour. His speed in still water being 4 miles per hour, how far will he go in 1 hour if he rows (i.) down stream, (ii.) up stream?

Let O (Fig. 46) be the man's starting point, $B'OB$ the direction of the river.

Then in 1 hour the water that was at O will be at a point A, 1 mile down stream, and, if the boat had been allowed to drift, it would be at A.

Fig. 46.

In the first case the man pulls his boat 4 miles *through the water* down stream in the hour, and will therefore be at B where AB is 4 miles.

Therefore the whole distance travelled in 1 hour is OB, which $= 1+4$ or 5 miles.

In the second case the man pulls his boat 4 miles up stream through the water, and will therefore be at B' where AB' is 4 miles.

Therefore the whole distance travelled in 1 hour is OB', which $= 4-1$ or 3 miles.

In this example the velocity of the boat through the water and the velocity of the stream are the *component velocities*, and the velocity of the boat relative to the banks or bed of the river is the *resultant velocity.*

If the proper signs are given to the component velocities, then for velocities in a straight line

resultant velocity = sum of component velocities.

We must now show how to find the resultant of two velocities not in the same straight line.

Example.—A sailor climbing the mast of a steamer in motion has two distinct velocities along different lines. This does not mean that he is moving in two lines at the same time; for this is impossible. What is meant is that the actual motion of the sailor (*i.e.* relative to the earth) is the resultant of two motions, viz., his motion up the mast relative to the steamer and the motion of the steamer relative to the earth.

64. THE PARALLELOGRAM OF VELOCITIES.—If two component volocities are represented completely by two straight lines drawn from a point and the parallelogram which has those straight lines for adjacent sides is completed, the resultant velocity is represented completely by the diagonal of the parallelogram passing through the point.

Let the component velocities be represented by the lines OA, OB. (Fig. 47.)

Complete the parallelogram $OACB$ and join OC. Then OC represents the *resultant* or *actual* velocity of the body both in magnitude and in direction.

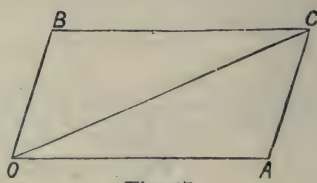


Fig. 47.

It is difficult to prove the parallelogram of velocities experimentally. A simple theoretical proof is:—Suppose the body only to have the velocity represented by OA (Fig. 47). Then, at the end of 1 second it would have been at A. But at each instant it has had a tendency to move parallel to OB with a velocity that would have taken it to B in 1 second if the body had not also been moving with velocity OA towards A. The result is, therefore, that the body arrives at C by the path OC.

Note the similarity between this theorem and the Parallelogram of Forces (§ 33). There is also a corresponding theorem to the Triangle of Forces (§ 36). The graphic methods for determining the resultant of two velocities, or the components in specified directions of one velocity, are the same as for the corresponding propositions relating to forces (§§ 34, 37).

Summary.—Chapter VII.

1. *Motion* is change of position. *Velocity* is the *rate* of change of position. To specify a velocity completely it is necessary to state (i.) the *magnitude* of the velocity, (ii.) the *line of motion* of the body, (iii.) the *direction* along that line. If a velocity with a particular direction be called positive, then one with a contrary direction is called negative. (§§ 3, 54, 56.)

2. *The average or mean velocity* of a body A during a stated interval is the velocity of another body B which, moving uniformly, would pass over an equal distance in the same time. (§ 61.)

3. If the velocity be *uniform*, the *distance traversed* is equal to the *product* of the *velocity* and the *time*. Also, if the velocity be *variable*, the distance traversed is equal to the product of the **average** velocity and the time. (§§ 58, 59, 61.)

4. *The Parallelogram of Velocities.*—If two component velocities are represented completely by two straight lines drawn from a point and the parallelogram which has those straight lines for adjacent sides is completed, the resultant velocity is represented completely by the diagonal of the parallelogram passing through the point. (§ 64.)

EXERCISES VII.

1. Express the following speeds in feet per second:—

(i.) $2\frac{1}{2}$ ft. in $3\frac{1}{2}$ secs. (iii.) 5 yds. in 3 mins.

(ii.) 20 miles per hour. (iv.) x ft. in n secs.

(v.) x yds. in n hrs.

2. Compare the speeds of two bodies, one of which moves over 5 yds. in 7 secs. and the other over 20 ft. in 4 secs.

3. Express the speeds: (i.) 5 ft. per sec., (ii.) 32 mls. per hr., in cms. per sec.

4. Express, in the F.P.S. system of units, the velocity of a point on the equator, supposing the circumference of the earth to be 25,000 mls.

5. A body goes x ft. in n secs. How many hours will it take to go y miles?

6. A train 90 yds. long passes completely through a station 130 yds. long in $11\frac{1}{4}$ secs. At what rate was it travelling?

7. A stone falls through 48, 80, and 112 ft. in three consecutive seconds. What is its average speed?

8. The actual velocity of a man on the deck of a steamer is 7 mls. per hr. What is the speed of the steamer if the man is (i.) walking from bow to stern at 4 mls. per hr., (ii.) walking from stern to bow at 2 mls. per hr., (iii.) running from stern to bow at 10 mls. per hr.?

9. Express a velocity of 60 miles per hour in feet per second.

10. Compare the velocities of two bodies, one of which moves over 5 yds. in 10 secs., and the other of which moves in the same direction over 20 ft. in 5 secs.

11. Express a velocity of 1 ft. per sec. in centimetres per second.

12. Express a velocity of 1 cm. per sec. in feet per second.

13. A body is moving at a certain instant with a velocity of 18 kilometres per hour, and its velocity is uniformly diminishing at the rate of 5 cms. per sec. After how many seconds will the body come to rest?

14. The measure of the velocity of a body is 6 in the F.P.S. system. What is its measure when the units of time and space are (i.) a yard and a minute, (ii.) a mile and a hour, (iii.) a centimetre and a second?

15. A steamer moves up stream at 16 mls. per hr. through the water which is flowing out at the rate of 3 mls. per hr. What is the real velocity of a man on board when he walks from bow to stern at 4 mls. per hr.?

16. The average speed of a body during the first 3 secs. of its motion is 22 ft. per sec., and for the next 7 secs. it is 66 ft. per sec. Find the average speed for the whole time.

17. Two trains, A and B, moving towards each other on parallel rails uniformly at the rate of 30 mls. and 45 mls. per hr. respectively, are 5 mls. apart at a given instant. How far apart will they be at the end of 6 mins. from that instant; and at what distances are they from the first position of A?

18. With the aid of a scale and protractor calculate the resultant of velocities of 5 and 12 when the angle between them is (i.) 90°, (ii.) 30°, (iii.) 65°, (iv.) 135°, (v.) 165°.

19. The wind blows from a point between N. and E. The northerly component of its velocity is 10 miles per hour, and the easterly component is 36 miles per hour. Find the actual velocity.

20. A body has a velocity of 3 miles per hour due south, and one of $\sqrt{2}$ miles per hour due east. What is its actual velocity?

21. A ship is sailing due north at the rate of 4 ft. per sec., a current is carrying it due east at the rate of 3 ft. per sec., and a sailor is climbing a vertical mast at the rate of 2 ft. per sec. What is the velocity of the ship, and what the velocity of the sailor relative to the sea bottom?

22. Resolve a vertical velocity of 20 ft. per sec. into two components, one horizontal, the other inclined to the horizon at an angle of 60°.

23. Prove that, the greater the angle between two velocities, the less will be their resultant.

24. Two equal velocities have a resultant equal in magnitude to either of the velocities. Find the angle between their directions.

25. Two engines leave a station at the same time. One travels due north at the rate of 50 miles an hour; the other travels north-east at the rate of 35 miles an hour. Draw a diagram, and from the diagram find the velocity with which one engine is moving away from the other.

*CHAPTER VIII.

ACCELERATION. FALLING BODIES.

65. Acceleration.—DEFINITION.—**The acceleration of a body along a straight line is the rate of increase of its velocity along that line.**

When the velocity of a body is increasing it is said to be **accelerated.**

Thus, when a stone is let fall from the top of a cliff its motion is accelerated. The same thing happens when a train leaves a station.

The phrase *rate of increase of velocity* demands attention. A simple illustration will suffice to show its full meaning.

Example.—Suppose that a railway train starting from rest at a station acquires a velocity of 10 miles an hour at the end of the first minute, a velocity of 20 miles an hour at the end of the second minute, a velocity of 30 miles an hour at the end of the third minute, and so on. Obviously the speed is increasing at the rate of 10 *miles an hour* in each *minute*, or we can say

the *rate* of increase of velocity is 10 miles per hour per min.

66. Uniform and Variable Accelerations. — DEFINITIONS.—**The acceleration of a body is said to be uniform when the velocity increases by equal amounts in equal intervals of time, however small the intervals may be.**

When this is not the case the acceleration is said to be **variable.**

It is as necessary here as in the case of uniform velocity that the words *however small* should be present, and for similar reasons. An example will make this evident.

Example.—An eight-oared boat rowing 30 strokes to the minute starts in a race, and in 4 strokes attains a velocity of 12 miles per hour. During each of these strokes the velocity increases 3 miles per hour, *i.e.* the boat is moving 3 miles per hour faster at the *end* of each stroke than it did at the end of the preceding stroke. Thus the velocity increases by equal amounts (3 miles per hour) in equal intervals of time (2 secs.). But the greater portion of the increase of velocity takes place while the oars are in the water, and very little, if any, while the men are swinging forward. Therefore the acceleration of the boat is not uniform.

* See footnote on page 57.

67. Measure of Uniform Acceleration.—Acceleration, *when uniform, is* **measured by** *the* **increase of velocity in a unit of time.**

It follows that the *acceleration of a body moving uniformly is zero.*

The **F.P.S. unit of acceleration** is the acceleration of a body whose velocity increases in each second of its motion by 1 ft. per sec.

This is usually more briefly spoken of as an acceleration of **1 ft. per sec. per sec.**

The **C.G.S. unit of acceleration** is an acceleration of **1 cm. per sec. per sec.**

The phrase *an acceleration of* 1 *foot per sec. per sec.* and similar ones are a source of perplexity to the beginner. The repetition of the words *per sec.* appears to him quite unnecessary. But, if we replace the phrase by its uncontracted form, "an acceleration in which *in each second* the velocity is increased by a velocity of 1 ft. *per sec.*," the difficulty should disappear. If the latter *per sec.* of the contracted phrase were omitted, we should learn that a certain increase of velocity, 1 ft. per sec., took place, but we should have no information as to *how long it was in taking place* (*cf.* § 65 carefully).

Examples.—(1) If the velocity of a body at any instant is 5 ft. per. sec. and increases uniformly so that a second later it is 9 ft. per sec., the increase of velocity in 1 sec. is 4 ft. per sec., and therefore the acceleration is 4 ft. per sec. per sec.

(2) If a train starts from rest and its speed is increased at the rate of 10 miles per hour per minute, how long will it take to acquire a speed of 45 miles an hour?

To acquire a speed of 10 miles per hour takes 1 minute ;

$\therefore$ to acquire a speed of 45 miles per hour takes $\frac{1 \times 45}{10}$ or $4\frac{1}{2}$ mins.

68. Positive and Negative Accelerations. — If the velocity of a body is increasing, the acceleration is **positive;** if the velocity is decreasing, the acceleration is **negative.**

A negative acceleration is frequently called a **retardation.**

Examples.—(1) If in 1 sec. a velocity changes uniformly from 8 ft. per sec. to 11 ft. per sec., the acceleration is +3 ft. per sec. per sec. ; if, however, the velocity diminishes uniformly to 2 ft. per sec., the acceleration is −6 ft. per sec. per sec.

(2) A bullet is shot from a rifle with a speed of 1,600 ft. per sec. At the end of 3 secs. it is travelling with a speed of 1,050 ft. per sec. Find the average rate of decrease of speed.

The decrease of speed in 3 secs. is

(1500 − 1050) or 450 ft. per sec. ;

hence the average decrease in 1 sec. is

$(450 \div 3)$ or 150 ft. per sec.

69. To find the velocity after t secs. of a body moving with uniform acceleration of a ft. per sec. per sec.

Let the body at the beginning of the t secs. be at rest.

The acceleration is a, *i.e.* in *each* second of the motion the velocity of the body is increased by a velocity of a ft. per sec.

Therefore at the end of 1 sec. the velocity is a;

" " 2 secs. " " $2a$;

" " 3 secs. " " $3a$;

and so, generally,

at the end of t secs. the velocity is ta.

If v ft. per sec. is the velocity at the end of t secs., we have, therefore, $\boldsymbol{v = at}$ ..... **(3).**

Cor. If the body was initially moving at the rate of u ft. per second, then the final velocity v is equal to the initial velocity *plus* the increase in velocity,

i.e. $\boldsymbol{v = u + at}$ ..... **(4).**

Note.—The attention of the reader is drawn to the fact that the above formula is **true whatever units of time and distance we select, provided we keep to the same units throughout.**

Examples.—(1) A body has a uniform acceleration of 12 ft. per sec. per sec. Its initial velocity is 16 ft. per sec. What is its velocity after 10 secs. ?

Here, taking a foot and a second as our units, we have

$$a = 12, \quad u = 16, \quad t = 10;$$

$$\therefore \quad v = u + at = (16 + 12 \times 10) \quad \text{or} \quad 136 \textit{ feet per sec.}$$

(2) A body starts with a velocity of 50 cms. per sec., and is retarded 5 cms. per sec. per sec. When will it come to rest?

Taking a centimetre and a second as our units,

we have $u = 50, \quad a = -5,$

and the final velocity v is 0.

Substituting in $v = u + at$, we have $0 = 50 + (-5)t$;

$$\therefore \quad -50 = -5t; \qquad \therefore \quad t = 10 \textit{ secs.}$$

70. To find the distance passed over in t secs. by a body moving from rest with uniform acceleration a ft. per sec. per sec.

The initial velocity is 0; the final velocity is at (see § 69). Therefore, since the increase in velocity is uniform, the average velocity is $\frac{at}{2}$. The space described in time t, being equal to the product of the average velocity and the time, is therefore given by

$$s = \tfrac{1}{2}at \times t = \tfrac{1}{2}at^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5).$$

71. To express the final velocity in terms of the space passed over and the acceleration.

Let v equal the final velocity in ft. per sec.; then, by (3),

$$v = at.$$

Squaring, we get $v^2 = a^2t^2$;

but, by (5), $t^2 = \frac{2s}{a}$;

therefore, substituting for t^2 in the relation above, we get

$$v^2 = 2as \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6).$$

72. To find the distance passed over in t secs. by a moving body whose initial velocity is u ft. per sec. and whose velocity is being uniformly accelerated a ft. per sec. per sec.

Let v be the final velocity; then, by (4),

$$v = u + at.$$

Also, since the acceleration is uniform, the average velocity is equal to half the sum of the initial and final velocities,

$$i.e. \text{ equal to } \tfrac{1}{2}(u+v), \quad i.e. \text{ to } \quad u + \frac{at}{2}.$$

But the space passed over is equal to the product

average velocity × time,

$$i.e. \qquad s = \left(u + \frac{at}{2}\right) \times t$$

$$= ut + \tfrac{1}{2}at^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7).$$

73. To express the final velocity in terms of the initial velocity, the space described, and the acceleration.

We have, by (4),

$$v = u + at, \quad i.e. \quad v - u = at;$$

but, since $s = \frac{1}{2}(u+v)t$,

$$v + u = 2s/t.$$

Multiplying together the expressions for $(v+u)$ and $(v-u)$, we get $v^2 - u^2 = 2as$,

$$i.e. \quad v^2 = u^2 + 2as \ldots\ldots\ldots\ldots\ldots\ldots \textbf{(8)}.$$

Note that formulae (7) and (8) reduce to formulae (5) and (6) when $u = 0$.

Recapitulation.—The formulae we have obtained above for accelerated motion are—

(i.) for a body starting from rest,

$$v = at \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3),$$
$$s = \tfrac{1}{2}at^2 \ldots\ldots\ldots\ldots\ldots\ldots (5),$$
$$v^2 = 2as \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6),$$

and (ii.) for a body starting with an initial velocity u,

$$v = u + at \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4),$$
$$s = ut + \tfrac{1}{2}at^2 \ldots\ldots\ldots\ldots\ldots\ldots (7),$$
$$v^2 = u^2 + 2as \ldots\ldots\ldots\ldots\ldots\ldots (8),$$

where t is the number of seconds in the interval under consideration,

a is the acceleration,

v is the velocity at the end of this interval,

and s is the distance passed over in the interval.

It must be noticed that in any particular case of motion (of the kind we are here considering, viz., under uniform acceleration) a remains constant throughout the motion, while u is also a fixed quantity. Of the three variables t, v, s, each equation contains two. Equation (3) [or (4)] contains v and t, and therefore answers either of the questions

What will be the velocity in such and such a time?
In what time will the velocity be so and so?

Similarly, equation (5) [or (7)] answers:

In what time will the body describe such and such a distance?
What distance will the body describe in such and such a time?

Equation (6) [or (8)] answers:

What velocity will be acquired after describing such and such a distance?
What distance has been described when the velocity is so and so?

If two of these three variables are given, the corresponding equation gives us the acceleration a (supposing u is known, as, for instance, where a body starts *from rest*).

Caution 1.—In any example it is essential that a distinction be made between a case of *uniform velocity* and one of *uniform acceleration*. Formula (1), $s = ut$, is the *only* formula to be used for *uniform velocity*.

Caution 2.— Care should be exercised in choosing that formula which will give the required result most rapidly. The student should set down the values of the given quantities, and then ask himself which one of the four formulæ connects these quantities with the required quantity. This will be illustrated in the following examples.

Examples.—(1) A body moving from a state of rest with uniform acceleration describes 50 ft. in 5 secs. Find its acceleration.

Using $s = \frac{1}{2}at^2$, we get $50 = \frac{1}{2}a.25$.

$\therefore \quad a = 4$ ft. per sec. per sec.

(2) What distance is traversed by a body in 7 secs. if it starts with a velocity of 8 cms. per sec., and moves with an acceleration of 3 cms. per sec. per sec.?

Here $t = 7$, $u = 8$, $a = 3$, and s is required.

Using $s = ut + \frac{1}{2}at^2$, which is the only formula connecting the given quantities t, u, a and the required quantity s, we get

$$s = 8 \times 7 + \tfrac{1}{2} \times 3 \times 7^2 = 56 + \frac{3 \times 49}{2} = 129\tfrac{1}{2} \text{ centimetres.}$$

(3) A train, travelling at 40 miles per hour, is in 2 mins. brought uniformly to rest at a station by means of the brakes. At what distance from the station were the brakes applied?

Taking a mile and an hour as our units, $u = 40$, $v = 0$, $t = \frac{1}{30}$, and s is required.

The average velocity $= 20$.

$\therefore$ distance required $= 20 \times \frac{1}{30} = \frac{2}{3}$ mile.

Or we may, by using the formula $v = u + at$, first find a and then, from $s = ut + \frac{1}{2}at^2$, find s.

Thus $0 = 40 + \dfrac{a}{30}$; $\quad \therefore \quad a = -1200$ miles per hour per hour.

and $\qquad s = \frac{40}{30} \times \frac{1}{2}(-1200)\frac{1}{900} = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$ mile.

(4) A body moving with uniformly accelerated velocity has at one instant a velocity of 32 ft. per sec. After travelling 384 ft. its velocity is 160 ft. per sec. Find the acceleration.

Here $u = 32$, $v = 160$, $s = 384$, and a is required.

Using $v^2 = u^2 + 2as$, we get

$$160^2 = 32^2 + 768a; \qquad \therefore \quad 160^2 - 32^2 = 768a.$$

$$\therefore \quad a = \frac{192 \times 128}{768} = 32 \text{ (ft. per sec. per sec.)}.$$

74. Change of Units. — In § 62, examples were given in which a velocity was changed from one set of units to any other. We now give an example of the change of units in the case of an acceleration.

Example.—If the acceleration of a body is 55 in foot-second units, what is it in yard-minute units?

The velocity acquired in 1 minute by a body moving from rest with an acceleration of 55 ft. per sec. per sec.

$$= 55 \times 60 \text{ ft. per second}$$

$$= \frac{55 \times 60}{3} \times 60 \text{ yds. per minute}$$

$$= 66{,}000 \text{ yds. per minute.}$$

The acceleration is therefore 66,000 yds. per minute.

EXERCISES VIII.

1. A body moving from rest acquires in each second a velocity of 12 ft. per sec. Find (i.) the distance it passes over in the first 5 secs. of its motion; (ii.) its velocity 96 feet from the starting point.

2. When a particle is moving at the rate of 43 miles per hour what would be the velocity if estimated in feet and seconds? Suppose the velocity to be acquired uniformly in 11 secs., by how much is the velocity increased per sec.?

3. The acceleration of a body's velocity is denoted by 5, the units being feet and seconds. What fact is expressed by this number 5?

4. A body moves in a straight line under an acceleration of 6·5 ft. per sec. per sec. In what time from rest will it acquire a velocity of 130 miles per hour?

5. If the velocity of a body is increased uniformly in each second by 32 ft. per sec., by how many feet per second is its velocity increased in 1 min.?

If the velocity is 1920 ft. per sec., what is it in yards per minute?

6. The velocity of a body is increased uniformly in each second by 20 ft. per sec. By how many yards per minute will its velocity be increased in 1 min.?

75. The Acceleration due to Gravity.—It is a matter of observation that most bodies when removed from contact with other bodies fall to the Earth. This movement is due to **gravity,** *i.e.*, to the attractive force exerted by the Earth on the bodies.

There are however some bodies which, when free to move, *ascend* instead of descend. A balloon filled with gas is a well-known example of this. This upward motion is due to the fact that the force of gravity downwards is counteracted by another and a greater force *upwards*, due to the presence of the air, just as when a cork is placed in water the upward force due to the water displaced is greater than the weight of the cork.

If we take a lump of lead, a piece of paper, and a feather and release them from the hand at a height, say, of 5 ft. from the floor, they all fall to the ground in consequence of the force of gravity acting upon them. The lead reaches the ground very soon, the paper not quite so soon, while the motion of the feather is, in comparison, very slow. These differences are due to the presence of the air, as shown conclusively by an experiment first performed by Newton, and now called the **Guinea and Feather experiment.** Newton took a long glass tube (Fig. 48), about 3 inches in diameter, closed at one end. A guinea and a feather were then inserted, and the other end closed with an air-tight cap and a stop-cock. When the tube was inverted it was found that the times of falling were very unequal. The stop-cock was next attached to an air-pump and the air exhausted. The tube was then detached from the air-pump and the experiment repeated, and it was found that the guinea and feather moved side by side down the tube with equal velocities.

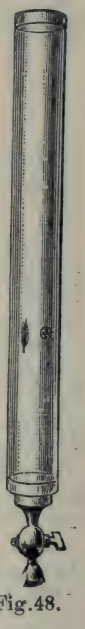


Fig. 48.

The same thing can be shown more simply without an air-pump, by the following experiments, which should be performed by the student before proceeding further:—

Exp. 30.—Take two equal masses of tin, about 1 oz. each, one in the form of a spherical ball and the other in the form of a very thin circular plate. Then drop them from the same height, holding the plate horizontally, and it will be found that the plate takes a longer time than the ball to reach the ground. This difference cannot be due either

to the *material* or to the *mass*, for they are the same for both bodies, but clearly arises from the fact that the plate has a larger amount of air to move out of the way. If now the plate be held *vertically*, so that it exposes only a small amount of surface to the air in the direction of its motion, and the experiment is repeated, no difference in the times of falling can be detected.

Exp. 31.—Take a small tin canister without the lid (*e.g.* a cocoa-tin), and in it place various objects, such as a coin, a feather, a piece of thin tissue paper, &c. Drop the canister from a height. All the objects will remain inside and will reach the ground together, showing that all are equally acted on by gravity.

Deductions.—(1) The resistance of the air causes bodies dropped simultaneously from the same height to reach the ground at different instants; (2) if the resistance of the air be removed, the bodies will reach the ground simultaneously.

It is thus clear that *in vacuo* (*i.e.*, in a space from which the air has been removed) all bodies move towards the Earth with velocities which (i.) are equal at the ends of equal intervals of time from rest, and (ii.) which increase at the same rate. In other words:

The acceleration due to gravity is the same for all bodies *in vacuo* at the same place on the Earth's surface.

The acceleration due to the force of gravity is usually denoted by the letter g. Its value varies slightly from place to place. The average value over **Great Britain** *at the sea level* can be taken as **32·19** (ft.-sec. units), or, more roughly, **32,** and in the C.G.S. system as **981.** Thus

$$g = 32 \text{ ft. per sec. per sec.} \quad \ldots\ldots\ldots \quad (9),$$

$$g = 981 \text{ cms. per sec. per sec.} \quad \ldots\ldots\ldots \quad (10).$$

By the statement $g = 32$ is therefore meant that when any body falls *in vacuo* its velocity increases in each second of its motion by a velocity of 32 ft. per sec.

NOTE.—The slight variations in the value of g are due:

(1) to the difference in the distance of the place of observation from the centre of the Earth: thus the Poles are nearer the centre of the Earth than the Equator; hence g at the Poles is greater than g at the Equator.

(2) to a force brought into play in consequence of the Earth's rotation about its axis. [See § 82 (4)]. For this reason also g at the Poles is greater than g at the Equator.

76. Formulae for Bodies falling Vertically. — The formulae we have already obtained for bodies moving with uniform acceleration, viz.,

$$v = at, \quad s = \tfrac{1}{2}at^2, \quad v^2 = 2as$$

and $$v = u + at, \quad s = ut + \tfrac{1}{2}at^2, \quad v^2 = u^2 + 2as,$$

can be used for bodies moving freely in a vertical line, provided that we replace

(1) a by g *if the body is moving downwards;*

For in this case the acceleration due to gravity tends to increase the velocity downwards, and is therefore positive. See § 68.

(2) a by $-g$ *if the body is moving upwards.*

In this case the acceleration diminishes the velocity.

Examples.—(1) A body is shot vertically upwards with a velocity of 48 ft. per sec.

(*a*) What velocity will it have after 1 sec. and where will it be?

(*b*) How far will it rise and how long will it take before it falls again to its starting point?

(*a*) Since the body is moving upwards, we have $u = 48$, $t = 1$, $a = -32$.

Using $v = u + at$, we get

$$v = 48 - 32 = 16 \text{ (ft. per sec.)}.$$

Using $s = ut + \frac{1}{2}at^2$, we get

$$s = 48 - 16 \cdot 1^2 = 32 \text{ (ft.)}.$$

(*b*) The velocity of the ball diminishes gradually, at last becomes zero, and then becomes downward.

Consider the *whole* upward motion:—

At the end of the upward motion the body is at rest; thus

$$v = 0;$$

also $u = 48$; $a = -32$.

$\therefore$ using $v^2 = u^2 + 2as$, we get

$$0^2 = 48^2 - 64s; \quad \therefore \quad s = 36 \text{ (ft.)}.$$

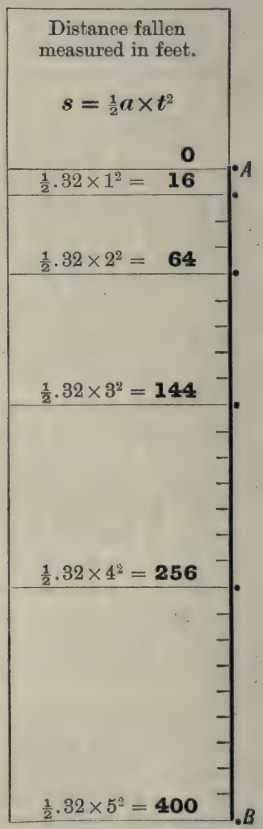


Fig. 49.

To find the time use $v = u + at$; then

$$0 = 48 - 32t; \quad \therefore \quad t = 1\tfrac{1}{2}.$$

Now consider the whole downward motion:—

At the highest point the body is at rest; thus

$$u = 0.$$

The body has 36 ft. to fall;

$$\therefore \quad s = 36; \quad \text{also} \quad a = 32.$$

$\therefore$ using $s = ut + \frac{1}{2}at^2$, we have

$$36 = 0 + 16t^2; \quad \therefore \quad t = 1\tfrac{1}{2} \text{ secs.}$$

$\therefore$ whole time up and down = 3 secs.

(2) From what height must a body fall to acquire a velocity of 40 ft. per sec.?

Here, using $v^2 = 2as$ and taking the positive direction downwards, we get $40^2 = 64s, \quad \therefore \quad s = 25$ (ft.).

(3) Mark on a scale the positions of a body falling from rest at the end of the first, second, third, fourth, and fifth seconds. Take as unit of length one division to represent 16 ft.

Tabulate the results as shown (Fig. 49).

Summary.—Chapter VIII.

1. Rate of change of velocity is called *Acceleration*. If this is uniform during any time, the average velocity is half the sum of the initial and final velocities. (§§ 65, 69)

2. If a body moves with uniform acceleration a,

starting from rest,		starting with velocity u,	
$v = at$	(§ 69),	$v = u + at$	(§ 69),
$s = \frac{1}{2}at^2$	(§ 70),	$s = ut + \frac{1}{2}at^2$	(§ 72),
$= \frac{1}{2}vt$	(§ 70),	$= \frac{1}{2}(u + v)\,t$	(§ 72),
$v^2 = 2as$	(§ 71);	$v^2 - u^2 = 2as$	(§ 73).

3. If g is the acceleration due to gravity,

$$g = 32 \text{ feet per sec. per sec.} \quad (\S\ 75)$$
$$= 981 \text{ centimetres per sec. per sec.} \quad (\S\ 75).$$

4. If a body is thrown vertically up or down with a velocity u, its motion is determined by the formulæ

$$\left.\begin{aligned} v &= u + at \\ s &= ut + \tfrac{1}{2}at^2 \\ v^2 - u^2 &= 2as \end{aligned}\right\} \quad (\S\ 76),$$

provided we substitute $a = +g$ or $-g$, according as the downward or the upward direction be taken as positive.

EXERCISES VIII. (*concluded*).

[In these examples g is to be taken as 32, except where a different value is expressly mentioned.]

7. How far will a body fall from rest in 10 secs., and what will be its velocity at the end of that time?

8. Find the velocity acquired by a body that falls freely from rest for $\frac{1}{4}$ sec., and the distance it describes in that time.

9. A body is thrown upwards with a velocity of 96 ft. per sec. After how many secs. will it be moving downwards with a velocity of 40 ft. per sec.?

10. A body is thrown upwards with a velocity of 110 ft. per sec. After how many seconds will it be moving downwards with a velocity of 60 ft. per sec.?

What supposition is implied in your answer as to the medium in which the body moves?

11. A rifle bullet is shot vertically downwards at the rate of 400 ft. per sec. How many feet will it pass through in 2 secs., and what will be its velocity at the end of that time?

12. From what height must a stone fall to acquire a velocity of 60 miles per hour?

13. It is a rainy day and I observe at the railway station that when a train enters the station and stops the water shoots off the front ends of the roofs of the carriages. I also observe that when the train starts the water streams off the back ends of the roofs. Explain this.

*CHAPTER IX.

FORCE AND THE LAWS OF MOTION.

In Chaps. I.–IV. we have discussed the action of *forces which are in equilibrium* and in Chaps. VII. and VIII. the *movements of bodies without any reference to the causes producing those movements.* We are now going to unite the two studies and consider the *relation between force and motion.*

77. Momentum.—If a person takes up a stone from the ground and holds it in his hand, he is conscious that he has to exert his muscles in order to support the stone. The *effort* he makes in doing so is small, if the stone is small.

Suppose now the stone be taken to a height of 10 ft. and dropped into the man's hand: he will feel that he has to make a greater effort than before to prevent the stone dropping to the ground, though it is the *same* stone he is trying to support.

Suppose, further, the stone is dropped from a height of 100 ft.: it will acquire a much greater velocity than it did in the second case, and it is probable that the greatest effort the man could put forth would not enable him to stop the stone.

It is thus clear that the effort required depends for one thing upon the velocity of the body, and increases with it.

Now suppose we try to stop in succession a cricket ball and a cannon ball of the *same size*, and moving with the *same velocity*. The effort required in the case of the cannon ball will be much greater than that in the case of the cricket ball, and this is due to the fact that the cannon ball is of greater mass.

The effort therefore depends upon the mass of the body as well as upon its velocity.

This ability possessed by a body in motion of overcoming resistances and of exerting force upon any body which it

strikes is summed up in the term **momentum,** which may be defined as follows :—

The **momentum** of a moving body is a **property** it possesses **by virtue of its mass and its velocity** conjointly, *and is measured by the product of its mass and its velocity.*†

Thus **momentum = mass × velocity** ..... **(11).**

Additional Illustrations.—(1) If a very large weight were placed upon the top of a pile which is to be driven into the ground, the weight might lie there for ever and yet not produce the required effect. What a heavy *weight* cannot do is, however, accomplished by the *momentum* of a much smaller body. An iron weight is hauled up with pulleys to a height of, say, 10 ft., and then allowed to fall, and by its momentum when it reaches the pile it drives the pile into the ground.

(2) The terrible effects witnessed in a cyclone or hurricane are due to the great momentum of the moving air. Though the mass is small, the velocity is very great, and consequently the momentum is large.

(3) To drive a nail into a piece of wood the momentum of the hammer is used; and a door may be forced open by the momentum of the body of a man.

78. Units of Momentum.—The **unit of momentum** is the momentum of unit mass moving with unit velocity.

The **F.P.S. unit of momentum** is therefore the momentum possessed by the mass of **1 lb.** *moving with a velocity of* **1 ft. per sec.** This is frequently called a **poundem.**

The **C.G.S. unit of momentum** is the momentum possessed by the mass of **1 gm.** *moving with a velocity of* **1 cm. per sec.** It is proposed to call this a **bole.**

Example.—Compare the momentum of a train of 40 tons mass, moving at the rate of 60 miles an hour, with the momentum of a cannon-ball of mass 112 lbs., moving at the rate of 800 ft. per sec.

Here mass of train = 40 tons = 89,600 lbs.,

and velocity of train = 60 miles an hour = 88 *feet per sec.*;

$\therefore$ momentum of train = mass × velocity = (89,600 × 88) *poundems.*

In the same way the momentum of the cannon-ball

= mass × velocity = (112 × 800) poundems;

$$\therefore \frac{\text{momentum of train}}{\text{momentum of ball}} = \frac{89{,}600 \times 88}{112 \times 800} = 88,$$

or the momentum of train = 88 times that of the ball.

† The words in italics must be included in the *definition* in order to distinguish *momentum* from *kinetic energy*, which is a property of a moving body dependent upon the mass and the *square of the velocity.* (See § 102.)

79. Change of Momentum.—If a body of mass m is moving with a velocity u, its momentum is mu.

If after t secs. its velocity has increased uniformly to v, its momentum has increased to mv, and therefore the change of momentum is

$$mv-mu \quad \text{or} \quad m(v-u).$$

This change takes place in t secs.

$\therefore$ **the rate of change of momentum** $= m(v-u)/t$.

But $v = u+at$, or $\dfrac{v-u}{t} = a$; *i.e.* $\dfrac{m(v-u)}{t} = ma$;

$\therefore$ **the rate of change of momentum** $= ma$ ..... **(12).**

80. The Laws of Motion. — We can now enunciate and explain the Laws of Motion, on which the whole science of Dynamics rests. Newton was the first to set forth these laws in a systematic manner, and in consequence they are usually termed Newton's Laws of Motion.

FIRST LAW OF MOTION.—Every body continues in a state of rest or of uniform motion in a straight line except in so far as it is compelled to change that state by external forces acting upon it.

SECOND LAW OF MOTION.—The rate of change of momentum of a body is in proportion to the external force acting upon it, and takes place along the line of action of the force in the direction in which the force acts.

THIRD LAW OF MOTION.—To every action there is an equal and opposite reaction.

81. Indirect Proof of the Laws of Motion.—It is impossible to give a direct mathematical *proof* of these laws. Many phenomena can be cited *tending to show* that the Laws of Motion are true, but the ultimate ground for believing the statements contained in them is as follows. The whole science of Astronomy, which deals with the intricate movements of the heavenly bodies, is built up on the Laws of Motion. The positions of these bodies at any moment are calculated and foretold years in advance, and the predicted positions are found to correspond most minutely with the actual observed positions. Now it is inconceivable that this could be the case if there were any error in the principles on which the calculations depend. We are thus forced to the conclusion that these principles, viz. the Laws of Motion, are true.

82. First Law of Motion.—*Every body continues in a state of rest or of uniform motion in a straight line, except in so far as it is compelled to change that state by external forces acting upon it.*

There are three statements contained in this law—

(1) A body at rest will remain at rest unless some external force acts upon it.

(2) A body moving along a certain straight line and in a certain direction with a certain speed will continue to move as above specified so long as no external force acts upon the body.

(3) If a body is neither at rest nor moving uniformly in a straight line, it is being acted upon by some external force. This fact, though not directly stated, is implied in the law.

The following illustrations tend to confirm our belief in this law.

Illustrations.—(1) If a stone be made to slide along the surface of a road, it will speedily be brought to rest. If we try the same experiment on ice, the stone will go much further, and will be moving for a longer time. If the ice be very smooth, the stone will continue to move for a much longer time and over a much greater distance, while the velocity will be still more slowly diminished. It is therefore not unreasonable to suppose that, if we could get rid of all the forces tending to stop the stone, among which are those caused by the roughness of the ice and the resistance of the air, the stone would move on for ever with uniform velocity.

(2) A top when spun in air will not spin for more than 3 or 4 minutes, but, if we perform the experiment in a vacuum, so that we get rid of the resistance of the air, the top can be made to spin in the condition called "sleeping" for more than two hours.

(3) The tendency of any body to continue moving at the same rate in the line in which it was moving is seen in the fact that when a horse ridden by a man stops suddenly the rider is in danger of being pitched over the horse's head. On the other hand, if the horse suddenly increases his speed, the rider will fall off behind unless he has a firm grip on the saddle.

(4) In a roundabout each rider experiences a tendency, at every point of his path, to move, not in the circle in which he actually does move, but in the direction of the *tangent* to the circle at the point, *i.e. to continue to move along the line* in which he was moving at the instant considered. He would do so did he not hold fast to some portion of the roundabout.

Exp. 32.—Suspend a fairly heavy weight—about 30 lbs.—and make a sudden attempt to pull it aside by means of a fine thread. The thread will break.

Deduction.—Force must be used to change a body's state of rest, and the body's resistance to a change of state of rest or motion is considerable.

83. Inertia.—A body cannot, *in virtue of forces within its own material*, change the state of rest or of uniform motion in a straight line of itself as a whole. The condition it is in at any instant is the condition it was in 5 minutes previously, unless some external force has acted upon it,

This incapacity of a body to act upon itself is called **Inertia.**

The First Law of Motion therefore affirms that all bodies have inertia, and consequently it is sometimes called the **Law of Inertia.**

84. It is convenient to take the Third Law before the Second Law.

Third Law of Motion.—*To every action there is an equal and opposite reaction.*

Whenever any body A exerts a force upon a body B, then by this law B exerts an equal force upon A along the same straight line and in the opposite direction along that line.

Two such forces taken conjointly constitute a *stress*.

Illustrations.—(1) A book lying upon a table presses the table downwards with a force P, say, and the table presses the book upwards with a force Q, which is equal to P, and in the opposite direction.

If the pressure of the table were the only force acting on the book, it would not remain at rest. The student must be careful not to confuse either of the forces, one of which acts on the table and the other on the book, with the *second* force acting upon the book. This is the force with which the Earth attracts it, and is termed the *weight* of the book. This and the reaction Q of the table counteract each other, and consequently the book remains at rest.

(2) A magnet attracts a piece of steel with a certain force, but at the same time the magnet is attracted, along the same line, towards the steel with a numerically equal force. *Each* of the two bodies will, if free to do so, move towards the other as a result of the attractive force upon it.

(3) When a boat is pushed off with a certain force from the side of a floating barge, there comes into play a numerically equal but oppositely directed force, which drives the barge from the boat. The velocity of the barge increases very slowly because its mass is very large, and consequently the acceleration the force sets up in it is very much less than the acceleration the equal force sets up in the smaller mass of the boat.

(4) When a horse is starting a canal boat, it pulls the boat forward with a certain force, and consequently the boat exerts, by means of the connecting rope, a force on the horse *backwards*. *If this were the only force acting on the horse*, he would move backwards, and not forwards, as is actually the case. What, then, is the cause of the horse's forward movement? It is to be found in the action and reaction going on between the horse's hoofs and the ground. The hoofs press the ground backwards, and in consequence the ground presses the hoofs of the horse, and through them the horse himself, forwards. The two forces acting on the horse are therefore

(i.) the reaction of the ground, which tends to drive the horse forwards;

(ii.) the tension of the rope, which tends to draw the horse backwards;

and it is the excess of the first over the second that gives rise to the forward movement of the horse.

85. Second Law of Motion.—*The rate of change of momentum of a body is in proportion to the external force acting upon it, and takes place along the line of action of the force in the direction in which the force acts.*

This law is capable of experimental proof; but, as the experiments are somewhat difficult, we do not give them.

If therefore the external force is constant, the rate of change of momentum of the body is constant. But momentum is equal to the product of the mass of the body and its velocity; therefore the rate of change of velocity is constant, or, in other words, the body is moving with a uniform acceleration. Hence

(i.) *A constant force produces in a given mass a constant acceleration which is proportional to that force.*

A falling body furnishes a familiar example. The constant force is the weight of the body, and, as we have already seen, the body moves with constant acceleration.

(ii.) *If different forces operating on different masses produce the same acceleration, the forces are proportional to the masses.*

If then a force F produces in a body of mass m an acceleration a, **F varies as ma.**

We can therefore put $F = kma$,

where k is some constant whose value will depend upon the way in which we measure the force.

The student should notice that no assumption is made as regards the units of mass, force, &c., in this article. Hence the results are true, whatever the units may be; and therefore we may assume any units which may happen to be convenient in any experiment in which these results are used.

Let us now *choose* as our **unit of force that force which creates in a unit mass the unit of acceleration.** Such a force is called the **absolute unit of force.**

We then have $F = 1$ when $m = 1$ and $a = 1$;

$$\therefore \quad 1 = k \times 1 \times 1, \quad i.e. \quad k = 1.$$

$$\left.\begin{aligned} &\therefore \quad F = ma \\ \text{or} \quad &\textbf{force} = \textbf{mass} \times \textbf{acceleration} \end{aligned}\right\} \ldots\ldots\ldots \textbf{(13).}$$

This equation is of the utmost importance.

86. Absolute or Dynamical Units of Force.—

The British absolute (or F.P.S.) unit of force is therefore **that force which when acting upon the mass of 1 lb. gives it an acceleration of 1 ft. per sec. per sec.**

This form is termed the **poundal.**

It will be shown in § 91 that the poundal is nearly equal to the *weight* of half an ounce.

The C.G.S. unit of force is that force which when acting upon the mass of 1 gm. gives it an acceleration of 1 cm. per sec. per sec.

This force is termed the **dyne.**

The poundal and the dyne are termed *absolute* units of force because they are independent of the variations in the value of g mentioned in § 75.

From formula (13), we have

$$a = \frac{F}{m}.$$

Therefore, in the F.P.S. system,

$$\textbf{acc. (in ft. per sec. per sec.)} = \frac{\textbf{force (in poundals)}}{\textbf{mass (in pounds)}} \quad \textbf{(14)};$$

and, in the G.C.S. system,

$$\textbf{acc. (in cms. per sec. per sec.)} = \frac{\textbf{(force in dynes)}}{\textbf{mass (in grammes)}} \quad \textbf{(15)}.$$

Examples.—(1) A force of 5 poundals acts upon a mass of 6 oz. What acceleration does it produce?

Here $F = 5$ poundals, $m = 6$ oz. $= \frac{3}{8}$ pound;

$$\therefore \quad a = \frac{\text{force (in poundals)}}{\text{mass (in pounds)}} = 5 \div \tfrac{3}{8} \text{ or } 13\tfrac{1}{3} \text{ feet per sec. per sec.}$$

(2) A mass of 2 kilogrammes is acted upon by a force which gives it an acceleration of 10 cms. per sec. per sec. What is the force?

Here $a = 10$, $m = 2$ kilogrammes $= 2000$ grammes.

$$\therefore \quad F = ma = (2000 \times 10) \text{ or } 20{,}000 \text{ dynes.}$$

EXERCISES IX.

1. Express in poundems the momentum of 6 lbs. moving at the rate of 4 ft. per sec.

2. Two bodies move with constant velocities, one describing 36 miles in 1 h. 20 m., the other 55 ft. in $1\frac{1}{4}$ sec. Compare the two velocities, or express each of them as a velocity of so many feet a second. If the former body weighs 50 lbs. and the latter 72 lbs., compare their momenta.

3. A particle moves in a straight line, and for any second of its motion the velocity at the end of the second is 6 ft. a sec. greater than the velocity at the beginning of the second. What is the acceleration of the velocity? What inference can be drawn as to the force which acts on the particle?

4. There are two bodies whose masses are in the ratio of 2 to 3, and their velocities in the ratio of 21 to 16. What is the ratio of their momenta? If their momenta are due to forces P and Q acting on the bodies respectively for equal times, what is the ratio of P to Q? State the general principles which justify your answers.

5. A body whose mass is 12 lbs. is found to gain a velocity of 15 ft. a second when acted on by a constant force (P) for 3 secs. Find the number of poundals (or British absolute units of force) in P. What ratio does P bear to the force exerted by gravity on the body?

6. A force F acting on a mass of 5 lbs. increases its velocity in every second by 12 ft. a sec.; a second force F_1 acting on a mass of 28 lbs. increases its velocity in every second by $7\frac{1}{2}$ ft. a sec. Find the ratio of F to F_1.

7. Express in C.G.S. units the momentum of (i.) 8 gms. moving at the rate of 7 cms. per sec., (ii.) 3 kgms. moving at the rate of 3600 metres per hour.

8. Equal forces act for the same time on two bodies A and B, the mass of the first being four times that of the second. What is the relation between the momenta generated by the forces?

87. The relation between Mass and Weight.—That a relation exists between the mass of any body and its weight is evident, and we shall now proceed to find this relation and to show that weight may be used to measure force. In all that follows the student should be careful to remember that *weight is a force*, and that *mass* is not a force, but the *quantity of matter* in a body.

We should, however, first of all notice that it is only owing to the fact that the Earth attracts all bodies that what is known as weight has any existence. If the Earth ceased to exercise its attractive force, bodies would no longer have weight. Yet their masses would in no way be affected. It

is true that we should no longer be able to estimate the mass of a body by "weighing it," but it would be erroneous to imagine that its mass had also disappeared. The body would still offer resistance to the passage of any other body through its material, and, if it were placed on a smooth table, it would require precisely the same force to give it a certain acceleration as it would have done before the cessation of the Earth's attraction.

Again, if a stone were removed from the surface of the Earth and placed near its centre, it would not be attracted by the Earth in any direction and would not, therefore, have any weight. Yet, if it were in motion, it would require the same force to stop it as would have been needed for the same purpose at the surface of the Earth. Thus its mass has not been affected by its removal to the Earth's centre, though its weight has disappeared.

Let us apply the formula $F = ma$ to the case of a body which is allowed to fall freely *in vacuo*.

Let the mass of the body be m lbs. The *only* force acting upon it is its weight: let this be W *poundals*.

From § 75 we know that the acceleration of the body due to its weight is g ft. per sec. per sec., where g is approximately 32·2.

Substituting this in $F = ma$, we get

$$W = mg,$$

i.e. **wt. of body (in poundals) = mass (in lbs.)** $\times g$ **...(16),**

where $g = 32{\cdot}2$.

In a similar manner, for the C.G.S. system, we get

wt. of body (in dynes) = mass (in gms.) $\times g$ **...(17),**

where $g = 981$.

88. The weights of two bodies at the same place are in proportion to their masses. — Let W, w be the weights of the two bodies in poundals and dynes, as the case may be; and let their masses be M and m.

Then, since both bodies have the same acceleration g when allowed to fall freely, we have, by (16) or (17),

$$W = Mg,$$

$$w = mg,$$

where g is the value of the acceleration due to gravity at the particular place.

$$\therefore \quad \frac{W}{w} = \frac{Mg}{mg} = \frac{M}{m};$$

which proves the proposition.

89. The weight of a body is not constant.—In § 75 it was pointed out that the value of g is variable and depends upon the place of observation. Now the mass of a body is constant so long as no portion of it is taken away, though it may be changed in appearance. It therefore follows from

$$W = mg$$

that the weight of a body varies with the value of g, and is therefore different at different places of the Earth's surface.

The student should now revise § 9 on weighing by a **pair of scales** and a **spring balance.**

90. Gravitation or Statical Units of Force.—For everyday and engineering purposes the *weight of a pound* and the *weight of a gramme*,† *i.e.* the weights of the units of mass, are taken as the units of force in England and France respectively. In § 89 we saw that these forces are not constant, depending as they do upon the value of g, and consequently they are unsuitable for scientific purposes. From their connection with the Earth's attraction they are called **Gravitation Units of Force.**

Caution I.—Very frequently forces when measured in this way are spoken of as "a force of 6 lbs." or "a force of 2 tons" when by these phrases are meant *forces equal to the* **weights** *of* 6 lbs. and 2 tons. Remember that 6 lbs. and 2 tons are *masses*, not *forces*. Similarly, "a force of 4 gms." means "a force equal to the weight of 4 gms." These may be abbreviated into "a force of 6 lbs. wt.," &c.

Caution II.—The formula $F = ma$ is *not* true when the force is expressed in *lbs. wt.* or *gms. wt.*, but only when in *poundals* or *dynes*, and **all forces must be expressed in poundals or dynes before $F = ma$ is used.** We shall now find the relation between

(i.) the poundal and the pound weight,

(ii.) the dyne and the gramme weight.

† As the wt. of the gramme is small, the wt. of the kilogramme (1000 grammes) is more commonly used.

91. To express the gravitation unit of force (lb. wt. or gm. wt.) in terms of the absolute unit of force (poundal or dyne). In § 87 we learnt that

wt. of body in poundals = mass of body in lbs. $\times g$,

where $g = 32{\cdot}2$ approx.

If the body be *one* pound mass, we get

wt. of 1 lb. when expressed in poundals $= 1 \times g$,

i.e. **1 lb. wt. = g poundals** ..... **(18)**,

where $g = 32{\cdot}2$ approx.

Similarly, **1 gm. wt. = g dynes** ..... **(19)**,

where $g = 981$ approx.

Thus, 1 poundal = wt. of $\frac{1}{32{\cdot}2}$ lbs. approx.

= wt. of $\frac{16}{32{\cdot}2}$ oz. „

i.e. **1 poundal = wt. of half an ounce** (nearly)...**(20)**.

Similarly, 1 dyne = wt. of a milligramme (nearly).

Since the unit of mass is g times the unit of weight, some engineers have proposed to make their system consistent by introducing a new unit of mass, called the *slugg*, which is equal to g lbs., *i.e.* 32 lbs.; but the result is not satisfactory.

92. Application to problems.—It is by far the safer plan in all examples in which the relation between the force acting upon a body and the acceleration caused by that force is used [$F = ma$] to work in absolute units, *i.e.* in poundals or dynes, in preference to lbs. wt. or gms. wt. If, therefore, the forces are given in gravitation units, *convert them first of all into absolute units* by means of formula (18) or (19), *i.e.* by *multiplying* by the corresponding value of g. Any forces that have to be calculated will then be obtained from the equations of motion in poundals or dynes. These may be, if desired, converted back into gravitation units by *dividing* by the corresponding values of g, viz. 32 and 981.

Examples.—(1) A body whose mass is 6000 cwt. is acted on by a force of 1 ton wt. How long will it take to acquire a velocity of 3 miles per hour?

Here the mass = 6000 cwt. = (6000×112) *lbs.*,

and the force = 1 ton wt. = 2240 lbs. wt. = (2240×32) *poundals*.

$\therefore$ by $F = ma$, we have

$$2240 \times 32 = 6000 \times 112 \times a\,;$$

$$\therefore \quad a = \frac{2240 \times 32}{6000 \times 112} = \tfrac{8}{75} \text{ ft. per sec. per sec.}$$

But $u = 0$ and $v = 3$ miles per hour $= \tfrac{22}{5}$ ft. per sec.

$\therefore$ from $v = u + at$, $\qquad \tfrac{22}{5} = 0 + \tfrac{8}{75} . t\,;$

$$\therefore \quad t = \frac{22 \times 75}{5 \times 8} \text{ secs.} = \frac{165}{4} \text{ secs.} = 41\tfrac{1}{4} \text{ secs.}$$

(2) What distance is described in 2 secs. by a mass of 1 kilogramme when acted on by a force of 8 gms wt.? (Take $g = 980$.)

Here $m = 1$ kilog. = 1000 *gms.*, $F = 8$ gms. wt. = (8×980) *dynes*.

$\therefore$ by $F = Ma$, we get

$$8 \times 980 = 1000 \times a\,;$$

$$\therefore \quad a = \tfrac{784}{100} \text{ cms. per sec. per sec.,}$$

and $$s = \tfrac{1}{2}at^2 = \tfrac{1}{2} \cdot \tfrac{784}{100} \cdot 2^2 \text{ cms.} = 15{\cdot}7 \text{ cms.}$$

Summary.—Chapter IX.

1. Momentum of a mass m moving with a velocity u is measured by mu. (§ 77.)

2. The Laws of Motion. (§ 80.)

3. The magnitude of a force is proportional to the change of momentum it will produce in a unit of time. (§ 85.)

4. The absolute unit of force is that force which, acting on unit mass, creates in it unit acceleration. (§ 86.)

5. When a force F $\left(\begin{matrix}\text{poundals} \\ \text{or} \\ \text{dynes}\end{matrix}\right)$ acts upon a mass m $\left(\begin{matrix}\text{lbs.} \\ \text{or} \\ \text{gms.}\end{matrix}\right)$ and creates in it an acceleration a $\left(\begin{matrix}\text{ft. per sec. per sec.} \\ \text{or} \\ \text{cms. per sec. per sec.}\end{matrix}\right)$, then

$$F = ma. \qquad (\S\ 85.)$$

6. Hence, acceleration in ft. per sec. per sec.

$$= \frac{\text{force (in poundals)}}{\text{mass (in pounds)}}, \qquad (\S\ 86.)$$

and acceleration in cms. per sec. per sec.

$$= \frac{\text{force (in dynes)}}{\text{mass (in grammes)}}. \qquad (\S\ 86.)$$

7. The *weights* of different bodies produce in these bodies the same acceleration and are therefore proportional to the *masses*. (§§ 85, 88.)

8. The wt. of a body (in poundals) = mass (in lbs.) $\times g$, (§ 87.)
where $g = 32$ nearly.

9. The wt. of a body (in dynes) = mass (in gms.) $\times g$, (§ 87.)
where $g = 981$ nearly.

10. 1 lb. wt. $= g$ poundals, where $g = 32{\cdot}2$ nearly. (§ 91.)

11. 1 gm. wt. $= g$ dynes, where $g = 981$ nearly. (§ 91.)

EXERCISES IX. (*concluded*).

9. Convert the weights of (i.) 5 lbs., (ii.) 8 gms., (iii.) 3 oz., (iv.) 2 qrs., (v.) 7 kgms. into the corresponding absolute units of force.

10. Express (i.) 96 poundals as lbs. wt.

11. Express (i.) $245\frac{1}{4}$ dynes as gms. wt.

12. A force acting upon a certain mass gives it an acceleration of 147 cms. per sec. per sec. Compare the force with the weight of the mass. ($g = 980$.)

13. A stone of 4 lbs. mass is drawn up from the bottom of a cliff by means of a string with an acceleration of 3 ft. per sec. per sec. Find the tension of the string.

14. A body whose mass is 30 lbs. starts with a velocity of 40 ft. per sec. and is resisted by a constant force which stops it in 20 mins. Find the force.

15. If the pointer of a spring balance gives the same indication at different places on the Earth's surface, does it follow that the masses in the pan are equal? If not, why?

16. Define *momentum*, and say how it is measured. What is the momentum of a mass of 6 lbs. which has fallen freely for 4 secs.?

17. What is meant by the *inertia of matter*? State the three Laws of Motion and give illustrations of them. What reasons have we for believing these laws to be true?

18. State exactly the relation which holds good between the mass of a body, a force, and the acceleration of the velocity of the body produced by the force.

19. A force of 10 poundals acts upon a mass of 5 oz. Determine the acceleration.

20. Of which of the laws of motion are the following two cases of projection examples?—

(i.) A speck of mud flying from off a carriage wheel.

(ii.) A rocket.

21. What is the difference between the *mass* and the *weight* of a body?

22. A certain force acting on a mass of 10 lbs. for 4 secs. produces in it a velocity of 60 ft. per sec. Compare the force with the weight of 1 lb., and find the acceleration it would produce if it acted on 20 lbs.

23. A mass of 10 lbs. is acted upon by a constant force, and acquires a velocity of 20 ft. per sec. in 4 secs. Find the force.

24. What acceleration and what momentum will be created in a mass of a ton by a force equal to the weight of 1 cwt. acting for 6 secs.?

25. It is found that a body has its velocity increased by 7 ft. per sec. in any second of its motion; it is known that the body weighs 23 lbs. What is the magnitude of the force producing this acceleration? How many lbs. of matter would this force support against gravity in a place where $g = 32{\cdot}2$?

*CHAPTER X.

WORK, POWER, AND ENERGY.

93. The words "work," "power," and "energy" are used in Mechanics with certain technical meanings which are explained in this chapter. The student must distinguish very carefully between the scientific use of these words and their use in ordinary language.

WORK.

94. Work.—DEFINITION.—**Whenever a force acts upon a body in such a way that motion takes place, work is said to be done by the force.**

Examples.—A horse drawing a cart along a rough road; a bricklayer carrying bricks up a ladder; a man drawing water at a well or pump—in all these cases work is done.

But, unless motion takes place, no work is done.

When a man lifts a stone up from the ground, he does work; if he further *holds* it up at a certain distance from the ground he *exerts force*, but he does not do any work.

In the same way, the girders of the roof of a station exert force, but do not do work.

Thus, **work** *is done* **by a force** *when its point of application moves in the direction of the force.*

On the other hand, *when a body upon which a force acts moves* (owing to other causes) *in a direction opposite to that of the force*, **work** *is said to be done* **against the force.**

Examples.—When coal is hauled up a pit, work is done against the force of gravity. An engine in drawing a train on the level does work against the friction of the rails and axles, the pressure of the wind, &c.; if the train moves up hill, work is also done by the engine against the weight of the train.

95. Measure of Work.—*The work done by or against a force is measured by the* **product** *of the* **force** *(in units of force) and the* **distance** *(in units of distance) through which its point of application has moved* **parallel to the line of action of the force.**

If, therefore, the force is F, and the body moves through a distance s parallel to the line of action of the force, the

$$\text{work done} = Fs.$$

The phrase "the distance parallel to the line of action of the force" requires further explanation. If a man hauls a piece of marble up to the top of a house 90 ft. high, he does work against the weight of the marble through a distance of 90 ft., for this 90 ft. is measured vertically, *i.e.* in the line along which the weight of the marble acts If, however, he pulls the marble up an inclined plane of length 90 ft., s is not to be taken here as 90 when we are thinking of the work done against the weight. The weight acts *vertically*, and we must therefore inquire what is the *vertical* distance through which the marble has been raised. If the top of the plane is 55 ft. above the bottom, then work has been done against the weight of the marble through a distance of 55 ft.

96. Units of Work.—The **unit of work** is the work done by the unit force when its point of application is moved through unit distance parallel to its line of action.

Absolute units of work.—

The F.P.S. unit of work is the work done by a poundal acting through a distance of 1 ft. This is termed a **foot-poundal.**

The C.G.S. unit of work is the work done by a dyne acting through a distance of 1 cm. This is termed an **erg.**

Examples.—(1) A force of 5 poundals moves a body through 4 ft. along its line of action. How many foot-poundals of work does it do?

Here the force $F = 5$ poundals and the distance through which the body is moved along the line of action of the force is 4 ft.

$\therefore$ the work done $= Fs = (5 \times 4)$ or 20 ft.-poundals.

(2) A force of 9 dynes acting through a distance of 2 metres does (9×200) ergs, since 2 metres = 200 cms.

97. Practical Units of Work.—For ordinary purposes among engineers the foot-poundal and erg are too small, and the following units are used in the two systems:—

Foot-pound (ft.-lb.), which is the **work** *done by a force of* **1 lb. wt.** *acting through a distance of* **1 ft.**

This is equal to the **work** *done in raising a mass of* **1 lb.** *through a vertical distance of* **1 ft.**

Sometimes the **foot-ton** is used. This is the work done by a force of 1 ton wt. acting through a distance of 1 ft., and is therefore equal to 2240 ft.-lbs.

Since 1 lb. wt. equals 32 poundals, it follows that the work done by a force of 1 lb. wt. acting through a distance of 1 ft. is 32 times that done by a poundal acting through the same distance;

$$\therefore \quad \textbf{1 ft.-lb. = 32 ft.-poundals.}$$

Kilogrammetre, which is the **work** *done by a force of* **1 kilogram wt.** *acting through a distance of* **1 metre.**

This is equivalent to the **work** *done in raising a mass of* **1 kilogram** *through a vertical distance of* **1 metre.**

The kilogrammetre is equal to 98,100,000 ergs.

Examples.—(1) What work is done in raising a ton of coal from the bottom of a pit $\frac{1}{4}$ mile deep?

The force overcome = 1 ton wt. = 2240 *lbs.*,

and the distance along the line of action of the force = $\frac{1}{4}$ mile = 1320 *ft.*

$\therefore$ work done = Fs = (2240 × 1320) *ft.-lbs.* = 2,957,000 ft.-lbs.

(2) What work is done against gravity in drawing a mass of 1 ton up an incline $\frac{1}{2}$ mile long, rising 1 in 20?

The force overcome = the weight of the body = 2240 lbs. wt., and acts *vertically.*

The distance through which this force is overcome parallel to the line of action of the force is the height of an inclined plane $\frac{1}{2}$ mile long, rising 1 in 20, and therefore = $\frac{1}{20} \times \frac{5280}{2}$ ft. = 132 ft.

$\therefore$ work done against gravity = Fs = 2240 × 132, *i.e.* 296,000 ft.-lbs.

NOTE that the answers are in ft.-*lbs.* in these cases, because we expressed the forces in *lbs.* wt.

Power.

98. Power.—Definition.—Power is the rate of doing work. *The* **power** *of an "agent"* (*e.g.* an engine, a horse, or whatever does work) *is measured by the amount of work the agent is capable of performing per unit of time.*

Note.—Distinguish between force and power.

Work = force × length = power × time.

The F.P.S. dynamical or absolute unit of power is, of course, a rate of working of 1 foot-poundal per second. This unit is rarely used.

The C.G.S. dynamical unit of power is a power of 1 erg per second. This is too small for ordinary purposes, and, in, practice, the **watt,** which is equal to 10^7 units of power, *i.e.* 10,000,000 ergs per second, is used per second. It is principally used in electrical engineering.

99. Horse-Power.—*Gravitational Units of Power.*—The power of an engine is always measured in horse-power.

Definition.—A **horse-power** (H.P.) is a rate of working of

550 foot-pounds per second

or 33,000 foot-pounds per minute.

This unit of power was introduced by Watt, who estimated it as being the rate of working of a good horse, and it has been universally adopted by engineers as the unit of power. The power of an engine when expressed in horse-power is spoken of as the **horse-power** of an engine.

[Note that the horse-power is a gravitational unit.]

When engineers speak of *an engine of so many horse-power* —say a 10 H.P. engine—they mean an engine which is capable, under favourable circumstances, of working at 10 H.P.—*i.e.* performing 5,500 ft.-lbs. per sec. But such an engine might be worked more slowly, and might be used to perform, say, only 4,400 ft.-lbs. per sec. It would then be said the engine was working at $\frac{4}{5}$ of its full horse-power.

Examples.—(1) Find the H.P. of an engine which draws a railway train at 60 miles an hour against a resistance equal to the weight of 1 ton.

Here the engine moves 88 ft. per sec. against a resistance of 2240 lbs.-wt. Hence it performs

88 × 2240 ft.-lbs. per sec.

$$\therefore \text{ its H.P. is } \frac{88 \times 2240}{550} = 360 \text{ (approx.)}.$$

(2) A steam-pump raises 11 tons of water 15 ft. high every minute. What is its H P.?

Work done per minute $= 11 \times 2240 \times 15$ ft.-lbs.;

$$\therefore \text{ H.P.} = \frac{11 \times 2240 \times 15}{33000} = 11 \text{ (approx.)}.$$

EXERCISES X.

1. Define "work." Give an illustration to show how a falling body can do work.

2. How many practical units of work are done (i.) on 20 lbs. in falling 30 ft.; (ii.) by a man of 10 stone in walking upstairs to a height of 24 ft.; (iii.) by a force of 20 gms. wt. acting through a distance of $1\frac{1}{2}$ metres?

3. A man weighing 140 lbs. puts a load of 100 lbs. on his back and carries it up a ladder to the height of 50 ft. How many ft.-lbs. of work does he do altogether, and what part of his work is done usefully?

4. How is the work done by a force measured? If force is estimated in lbs. wt. and distance in ft., what is the unit of work commonly called?

5. Express 1400 ft.-lbs. (i.) in ergs; (ii.) in kilogrammetres.

6. A force of 1000 dynes acts through 2 metres. How many ergs of work are done?

7. Find the time which a man of mass 10 stones will take to climb a mountain 3,000 ft. high, if his power is 4200 ft.-lbs. per minute.

8. Find the H.P. of an engine which moves at the rate of 45 miles per hour, the mass of the engine and load being 100 tons, the frictional resistances being 20 lbs.-wt. per ton.

ENERGY.

100. Energy.—DEFINITION.—**The energy of a body is the quantity of work it is capable of doing.**

Energy refers to the *total quantity* of work the body can do, and implies nothing as to the time in which the work is done.

For example, a cyclist may be able on a particular day to put forth 2 million ft.-lbs. of work; but the rate at which the work is done may vary considerably. Riding at 12 miles per hour, he works, say, at the rate of 55 ft.-lbs. per sec., and his energy will be used up in about 10 hours, when he will be unable to proceed further. If he ride at a greater speed, he will have to work at a greater rate, and the same amount of energy will be used up in less time.

101. Kinetic Energy. — DEFINITION. — *The* **kinetic energy (K.E.)** *of a body is* **the quantity of work** *it is capable of doing* **by virtue of its mass and its velocity** *conjointly.*

Illustrations.—A cannon-ball in motion can do work by penetrating the sides of a ship, and the work it does is due to the fact that it has mass and is in motion.

A cyclist gets up speed before he comes to a hill, and his increased speed assists him in mounting the hill, and by means of it he does work against gravity.

Water in motion can turn a water-wheel and thus grind corn.

Exp. 33.—Energy of Visible Motion.—Paste a tightly stretched piece of tissue paper over a ring. On the paper place gently a bullet or a marble. The body is easily supported by the paper, and is not able to do sufficient work to depress the paper and break through it. Lift the body a few inches above the paper and let it fall; it depresses the paper and breaks through it. The bullet or marble must now be able to do work both against the resistance to depression and to breach which in the first case it could not do.

Deduction.—It follows, therefore, that the object has been able to do work in virtue of the fact that it was in motion at the time when it reached the paper. Thus it had **energy of visible motion** or kinetic energy.

102. Value of the Kinetic Energy of a body in terms of its mass and velocity.—

Suppose a body of mass in moving with velocity u is brought to rest by means of a force F acting against it in the line of motion. Suppose it moves a distance s before it is brought to rest, then the work done against the force $= Fs$.

Let a be the retardation caused by the force; then

$$F = ma.$$

Also, since the body is brought to rest, $v = 0$ in the equation $v^2 = u^2 - 2as$ (§ 73), and therefore

$$u^2 = 2as.$$

$$\therefore \quad s = \frac{u^2}{2a}.$$

Therefore the work done against the force, viz. Fs

$$= ma.\frac{u^2}{2a} = \tfrac{1}{2}mu^2.$$

But the body, being now at rest, is not capable of doing any more work, *i.e.* the total quantity of work a body can do is $\tfrac{1}{2}mu^2$;

$\therefore$ **the kinetic energy of the body** $= \tfrac{1}{2}mu^2$ ... **(21).**

Note that this is also equal to the work which a force would have to do if, acting on a body of mass m at rest, it gave it a velocity u.

Caution. — The value $\tfrac{1}{2}mu^2$ for the K.E. of a body is *measured* in absolute units of work, *i.e.* in *foot-poundals* or in *ergs*. Divide by g if necessary to express it in gravitational units.

Example.—Find the kinetic energy of (i.) a ball of 4 oz. mass moving at 60 ft. per sec.; (ii.) a train of 200 tons moving at 60 miles per hour. Express (i.) in ft.-poundals, (ii.) in ft.-lbs.

(i.) Here $m = 4$ oz. $= \frac{1}{4}$ *lb.*, $u = 60$ ft. per sec.;

$\therefore$ K.E. of ball $= \tfrac{1}{2}mu^2 = \tfrac{1}{2}.\tfrac{1}{4}(60^2)$, or 450 foot-poundals.

(ii.) Here $m = (200 \times 2240)$ lbs., $u = 88$ ft. per sec., $g = 32$.

$\therefore$ K.E. of train in ft.-lbs. $= \tfrac{1}{2}mu^2 = \tfrac{1}{2}\left(\dfrac{200 \times 2240}{32}\right) \times (88^2)$

$= 54{,}000{,}000$ foot-pounds (approx.).

103. The Equation of Energy. — If a force F acts through a distance s upon a mass m, and changes its velocity from u to v, the K.E. of the body increases from $\tfrac{1}{2}mu^2$ to $\tfrac{1}{2}mv^2$;

$\therefore$ the increase of K.E. is

$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

$$= \tfrac{1}{2}m(v^2 - u^2) = \tfrac{1}{2}m.2as = ma.s = F.s.$$

i.e. $\boldsymbol{Fs = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2}$ ..... **(22).**

or **work done by force on a body** $=$ { **increase of kinetic energy of body.**

This equation is called the **Equation of Energy.**

104. Potential Energy.—If a man throws a stone up to the top of a cliff, he does work against the weight of the stone. If the stone then lodges on the top of the cliff, it is in such a position that, if allowed to do so, *e.g.* by being pushed off the ledge, it will acquire a velocity, and therefore kinetic energy. By means of this kinetic energy it will be able to do the same amount of work as the man did in throwing the stone up.

We see then that the stone is capable of doing work merely by virtue of its position relative to the Earth, and to the work it can thus do is given the name *potential energy*.

DEFINITION.—**The potential energy (P.E.) of a body is the quantity of work it is capable of doing by virtue of its position.**†

In the case of gravitational potential energy, this position is reckoned relative to the Earth's surface, so that when the body is on that surface its potential energy is zero.

Exp. 34.—Transformation of Potential Energy into Kinetic Energy.—Hang a small weight by an inelastic string from a spring balance. Notice the position of the weight. Now raise the weight and let it fall. Notice that the weight descends below its first position, comes momentarily to rest at a definite spot, and then reascends, oscillating about its original position. The body, therefore, in stretching the string, does work against the tension of the string.

Deduction.—In raising the weight, we gave it *potential energy*. When it reached the position in which at first it rested it was in motion and, therefore, had *kinetic energy*. Thus the *potential energy* had been transformed into *kinetic*. When it reached its lowest position it had lost its *kinetic energy*, and also some more *potential energy*, but the spring now possesses *potential energy* due to *strain*.

Further Illustrations.—(i.) A pile driver before being allowed to fall has potential energy, which has been stored up in it by the men who pulled it up into position, and this energy is converted into kinetic energy when the pile driver is released.

(ii.) A lake up in the hills has potential energy. No work can be done by it, however, unless the water is allowed to run down, and thus acquire kinetic energy.

† Potential energy may also be due to a change in the relative positions of the particles of a body, as in the case of the main spring of a clock.

(iii.) When a watch spring is wound up work is done upon it against the elasticity of the spring. When permitted to do so the spring will emit this work again in turning the wheels of the watch as it tends to unwind itself. This also is a case of potential energy. In this case, however, energy is imparted to the body (*i.e.* the spring), not by raising it but by distorting it.

(iv.) Suppose two particles tied to a piece of elastic string and placed on a table so that the string is taut but not stretched. If the string be stretched and then set free, the particles move towards each other, *i.e.* the string employs force on the particles. Thus, in the stretched position, there was stored up in the string potential energy which was changed to kinetic energy of the particles when the string was released.

This case is analogous to that of two particles belonging to the same body which are separated by some means and which tend to come together again in virtue of their mutual attraction.

Or again it is analogous to the case of two metal balls oppositely charged with electricity and separated in spite of their mutual attraction.

We see then that a body may possess potential energy (1) when it is raised without distortion above the level of the Earth, (2) when its shape is distorted, or (3) when its particles are separated in spite of their mutual attraction.

If a body of weight W is at a height h above the ground,

its potential energy = Wh..... (23).

For this is the amount of work its weight would do if the body fell to the ground (§ 95).

Thus, if the mass of a body is M pounds, its weight $= Mg$ poundals, and its potential energy when at a height of h feet above the ground is equal to Mh ft.-lbs. or Mgh ft.-poundals.

Or, if its mass is M grammes, its weight is Mg dynes, and its potential energy when at a height of h cms. above the ground is equal to Mh gm.-cms. or Mgh ergs.

105. Equivalence of Kinetic and Potential Energy.—Consider a body of mass M placed at distance h above the ground. Its potential energy is Mgh and its kinetic energy is zero. If the body is now released, it falls to the ground, its velocity increasing from zero to v where $v^2 = 2gh$. (§ 76). On the ground the potential energy is zero and its kinetic energy is $\frac{1}{2}Mv^2$.

Since $Mgh = \frac{1}{2}M.2gh = \frac{1}{2}M.v^2$,

it follows that the potential energy of the body when stationary at the the height h is equal to its kinetic energy when the body just reaches the ground. During the fall the potential energy is gradually decreasing and the kinetic energy is gradually increasing. To show that, for a falling body, at every instant

potential energy + kinetic energy = a constant,

suppose that at the instant when the body has fallen a distance h_1 its velocity is v_1^2. We know from (§ 76) that

$$v_1^2 = 2gh_1.$$

The potential energy $= mg(h-h_1)$

and kinetic energy $= \frac{1}{2}mv_1^2$

$$\therefore \text{ P.E.} + \text{K.E.} = mg(h-h_1) + \tfrac{1}{2}m.2gh_1 = mgh$$

= potential energy of body before the fall

= kinetic energy of body at the instant it reaches the ground.

106. Energy can be transferred from one body to another.—If a billiard ball in motion strikes one at rest, the latter begins to move whilst the former moves on with speed diminished. Thus the ball which had no energy before the impact possesses energy after the impact, whilst the ball originally in motion loses part of its energy. It follows that energy has been transferred from the one ball to the other.

Impact is one method of transferring energy. Probably a more common way of bringing about such a transfer is illustrated as follows. Hold in the hand one end of a strong string to which a weight is attached. Move the hand to and fro in a horizontal line. No particle of the string moves, except to vibrate in a horizontal line, but the movement passes along the string until the particles near the weight and finally the weight itself are set in motion. Thus, without the transfer of any *material* from the hand to the weight, motion, and therefore kinetic energy, has been transferred by vibrating an intermediate medium. If a bundle of strings were all tied

to the weight and held by the hand, and if the hand were made to vibrate in the same way, exactly the same phenomenon would be witnessed. Thus, whether the intervening medium be as fine as a fine string or have a considerable section like the bundle of strings, energy may be transferred from one body to another without the transfer of matter. It is probably by means of the vibrations of some such intervening medium called the *ether* that the energy of the Sun is transferred to us and makes itself sensible as heat and light.†

107. Several forms of Energy. — There is reason to believe that the particles of every body, whether solid, liquid, or gas, are in perpetual vibration. Indeed the heat which bodies possess and emit is due to the vibration of their particles. Owing to their vibration these particles possess kinetic energy and it is assumed that what is called heat is due to this energy.‡

It is now firmly held that not only heat, but also light and sound are only forms of energy. Great experimenters have shown (1) that any one form of energy can be converted into any other, (2) that in such a conversion the quantity used up of the one is invariably equal to the quantity of the other created. It is not difficult to show the transformation from one form to another by simple experiment, but to show the strict equality between the quantities used and created is beyond the scope of this book. (See also § 112.)

108. Transformation of Mechanical Work into Heat.—

Exp. 35.—Hammer a piece of lead, saw wood, &c., and test the temperature of the lead, saw, &c., before and after the experiment. It will be found that the temperature has risen.

Exp. 36.—Rub a brass nail or button on a wooden seat, and notice its increase in temperature.

Deduction.—In both experiments the mechanical work expended in impact and friction has been transformed into heat.

Other instances of this transformation abound.

† See Part II., *Heat*, Chap. I., § 12, and Part III., *Light*, Chap. I., § 1.
‡ See also Part II., *Heat*, Chap. III., § 31.

109. When Air is compressed its temperature rises.—

Exp. 37.—Use a bicycle pump to inflate the tire or pump air into a closed bottle. Note that the pump becomes warm. Part of this is due to friction between the piston and the barrel; but the greater part is due to the heat developed in the air on compression.† This heat is equivalent to the work done in compressing the air.

110. Transformation of Electric Energy into Mechanical Energy.‡—

Exp. 38.—Suspend a pith ball by a dry silk thread between the charged knob of an electrical machine†† and a conductor connected to earth by a copper wire.

Notice that

(i.) the ball is attracted to the charged plate;

(ii.) it is then repelled until it touches the conductor;

(iii.) these motions are repeated a very considerable number of times.

When the pith ball is first suspended it has no kinetic energy. Immediately it is set free it moves, *i.e.* it takes up kinetic energy. What is the source of this energy? It cannot be the conductor, for that differs in no way physically from the suspended ball. The source, therefore, must be the electricity of the charged plate. This is confirmed by the facts that (i.) the first motion of the suspended ball is towards the plate, (ii.) the motion ceases when the plate originally charged ceases to give any sign of electrification when tested by the usual means.

Deduction.—We conclude, therefore, that the electrical energy with which the plate was originally endowed has, to some extent at least, been transformed into the kinetic energy of the particle.

Note.—Only a very small portion of the electrical energy, however, has been thus transformed, the major part of it has been discharged into the Earth through the earthed conductor.

---

† To test this point, work the pump with the nozzle open, making the same number of strokes at approximately the same rate: the pump becomes only slightly warm.

‡ See also Part V., *Electricity*, Chap. I. This section and the next may be kept till the second reading.

†† See Part V., *Electricity*, Chap. III.

111. Transformation of Electrical Energy into Heat.†—

Exp. 39.—Partly fill a fair-sized beaker with water. Solder a coiled piece of thin platinum wire to two thick copper leads coming from a battery of several cells through a key. Place the platinum coil in the beaker; place also therein a thermometer. Now press the key to let the current pass, and keep it down for some minutes.

Notice the rise of temperature indicated by the thermometer.

Deduction.—We have here, therefore, *heat energy* which did not previously exist; and the only possible source of it is the electrical current passing through the wire. This *electrical energy* is derived from the *chemical energy* of the constituents of the cell.

While the current is passing it will be found that a considerable quantity of heat is produced in the cells of the battery; this heat is partly due to the passage of the electric current through the cell, but is mainly due to chemical actions which are going on.

112. Conservation of Energy. — As we have already mentioned in § 107, and illustrated in §§ 105 and 108–111, it has been proved that whenever energy in one form disappears it is always replaced by an exactly equivalent amount of other forms of energy. Hence, when all the forms of energy are taken into consideration—

(1) *The total amount of energy in the universe is absolutely unalterable, i.e. no energy is ever created or destroyed.*

(2) *The various forms of energy may, though not always at will, be converted one into another.*‡

Energy may be transferred from one body to another, or transmuted from one kind to another, but the energy lost by one body is gained by other bodies, and *vice versa*.

The principle contained in the above propositions is known as the **Conservation of Energy.**

The numerical relation between heat energy and mechanical energy was established by Dr. Joule in 1843. **Joule's Law** states that 1 calorie†† of heat is equal to 42,000,000 ergs or 0·427 kilogrammetre of work; or that 1 water pound-degree Fahrenheit of heat†† is equal to 777 foot-pounds of work.

† See also Part V., *Electricity*, Chap. III.

‡ In nearly all operations some energy is wasted in friction, &c., going into the form of heat energy; as a rule, very little of this energy is recoverable by human agencies, and it is therefore said to be *dissipated*. In any store of energy that portion which can be used for actual mechanical work is often spoken of as the *available energy*.

†† See Part II., *Heat*, Chap. II., for definitions of these terms.

113. Application of the Conservation of Energy to Machines. — From the Principle of the Conservation of Energy it can be shown that no work is gained or lost when a perfectly frictionless machine† is used for raising weights or overcoming resistances. Hence, when the kinetic energy of a machine is unaltered, *the work done by the effort is always equal to the work done by the machine against the resistance.* This is called the **Principle of Work.**

Example.—A workman is raising a piece of granite of mass $\frac{1}{2}$ ton by means of pulleys and a windlass. The handle of the windlass describes a circle of radius 14 ins., and for every turn of the handle the granite rises 3 ins. If the machinery is frictionless, what force does the workman apply to the handle?

In one turn of the handle the machine raises the granite (a mass of 1120 lbs.) through $\frac{1}{4}$ ft. Hence the work done by the machine is $\frac{1}{4} \times 1120$ or 280 ft.-lbs.

In one turn the man moves the handle through $2 \times \frac{22}{7} \times 14$ or 88 ins.

Thus, if he exerts a force of F lbs., the work he does is $F \times \frac{88}{12}$ or $\frac{22F}{3}$ ft.-lbs.

Hence, by the above principle, $\frac{22F}{3} = 280$; whence $F = 38{\cdot}2$ lbs.

In actual practice the man would have to exert a much greater force owing to friction in the various parts of the machinery.

Taking, then, the **Principle of Work** for granted, we may use it to find the mechanical advantage of simple machines such as the lever and the inclined plane. Take the case of an inclined plane where the force acts parallel to the plane.

In Fig. 37 suppose originally that the particle of weight W is at the A, and that the effort P acting parallel to AC drags it slowly up to C. Assuming that there is no work lost in friction, and that the motion is so slow that there is no storing of kinetic energy, it follows from the Principle of Work that

work done by the effort P = work done against the gravity force W.

But work done by the effort $P = P \times AC$,

and work done against the gravity force $W = W \times BC$.

$$\therefore \quad P \times AC = W \times BC, \quad \text{or} \quad \frac{P}{W} = \frac{BC}{AC},$$

a relation identical with that obtained in § 41.

† In mechanics a **machine** is any contrivance through which force is transmitted so as to appear in a more convenient form at another point and overcome a resistance.

This principle is sometimes rendered in a popular saying as: "*What is gained in power is lost in speed*," meaning that, if the effort is small compared to the resistance, the effort must work through a large distance in order to move the resistance through a small distance; or that, if the machine is so arranged that the effort must move through a large distance in order to move the resistance through a small distance, then the effort need only be a small fraction of the resistance.

114. The Storing of Energy.—The preceding sections have dealt with the many different forms of energy and the methods by which one form can be transmuted into another.

Whether we look around us, above us, or below us we find on all hands huge stores of energy beneficently garnered by Nature, and it is the aim of the engineer to utilize these sources to their full advantage.

As a storehouse of energy the **Sun** is easily first. From the very beginning of things it has never ceased to radiate to the Earth energy in the form of heat and light. This energy supports life and growth in both animals and plants, loads the clouds with moisture, produces winds, and warms the atmosphere and the surface of the Earth and sea.

A **man** may be regarded as a secondary storehouse of energy. He is able to do work, but he needs constant replenishment in the shape of **food and air.** Food may therefore be looked on by man and beast as a store of energy.

A **plant** may be used in several ways: (*a*) it may constitute a food and so give life and strength to an animal, (*b*) it may constitute a fuel and be used for heating purposes or to drive an engine, or (*c*) it may die and rot on the ground, perhaps finally yielding coal or oil to be used as fuel by our remote descendants.

Coal is a great storehouse of chemical energy. When coal is burnt a part of this energy is used up in forming the combustion products, but the greater part is liberated as heat and light. It has been found that 12,000,000 foot-pounds of energy are liberated when one pound of coal is burnt. Kerosene oil has even a greater store, one pound yielding 20,000,000 foot-pounds of energy.

The heat developed in combustion of coal and other fuels may be used to boil water, and so to drive engines which may either draw heavy loads from one point of the country to another, put the machinery of a factory in motion, or drive a dynamo and so produce electrical energy.

At present, however, we are unable to prevent a very great waste of energy: even in our best engines only a small percentage of the energy produced by the burning of fuels is available for mechanical work.

Large stores of energy are also contained in **water located at high levels,** *e.g.* in mountain-lakes and rivers. This energy is made apparent at large waterfalls like Niagara and the Victoria Falls. Niagara has now been harnessed by electrical companies, and its output of energy now reaches over four million horse-power—formerly allowed to fritter away into waste heat at the bottom of the fall. Mill ponds store energy in a like manner in a smaller degree.

Energy may also be stored in an **uplifted weight** or a **wound-up spring.** For instance when a clock or watch is "wound up" potential gravitational energy is given to the weights or potential strain energy to the spring, and the energy thus stored drives the wheels. The pendulum or balance wheel only serves to regulate the output of energy by releasing a small fraction of the energy at each oscillation.

Now consider a locomotive engine drawing a train over a hilly country. Some of the hills are so steep that, if the train started from a state of rest in the valley below, the engine would not be able to surmount them. The line is, however, so engineered that before going up a steep hill the train may have a good run along the level or even down an incline so that the engine may "get up speed." At the bottom, therefore, the train contains a large amount of kinetic energy which helps to carry the engine up the hill. The case is similar to that of a person on a free-wheel bicycle, who, having stored up kinetic energy in the bicycle and himself, can safely take a rest until the energy has been nearly all used up in working against the frictional resistances of the rubbing parts of the bicycle and the road- and air-resistances.

A stationary engine cannot accumulate kinetic energy in itself as a whole, but an accumulator is provided for it

in the shape of a **fly-wheel,** the large wheel invariably seen on such an engine. The great function of the fly-wheel is to store energy and regulate its output. Take, for example, a one-cylinder engine, and let us suppose that the steam is only admitted at one end of the cylinder. At the first rush of steam the piston is driven out and sets the fly-wheel in motion. If there were no fly-wheel the piston would not return, but remain at one end of its stroke. The fly-wheel, however, has been set in motion, and part of its energy is used to send the piston back to the other end. The steam is again admitted, and the fly-wheel gains energy at every stroke until a high speed is attained. Suppose now the driving-wheel of the engine is connected to a circular saw used for cutting up trees. It is quite possible that the energy per minute required for this operation is more than is supplied per minute by the steam; hence without a fly-wheel the operation could not be performed.

With a fly-wheel, however, the engine gets up speed before the tree is fed to the saw and the operation is possible. By the time the cut is completed it is very likely that the speed of the wheel has greatly diminished, and the engine is therefore again allowed to get up speed before the next cut is performed.

Now consider an electric lighting station. The aim of the engineer is to keep his engines running at constant speed. In the day the demand is small, and the engineer has energy to spare; in the evening it is large, and he wants all the energy he can get. To solve the problem he takes a number of electric cells, called electric **storage cells,** and connects them to his dynamos. The extra supply of energy in the day is used to charge these cells with electrical energy. In the evening they are taken off and reconnected so that they help the dynamos.

Energy is frequently *transmitted* from one place to another by **hydraulic pressure,** *i.e.* by **water under pressure,** the pressure being supplied either by the head of the water behind it or by huge pumps at the pumping station. Pipes distribute the water throughout a city, where it operates lifts, pumps, and cranes. To provide for slack times, and also to act as a regulator, a huge **accumulator** is built at the pumping station. This is a huge heavily-weighted ram

fitting into a monster pump barrel. When the demand is slack, the pumping-engines pump into the accumulator and the ram rises. When the demand is large, the ram falls and thus supplements the engines.

Another storehouse of energy is the **atom.** Recent physical research shows the atom to be a storehouse of an immense supply of energy, which, however, we cannot tap at will. In a few cases—*e.g.* radium, thorium, uranium—we find atoms in a rapid state of transformation, energy being evolved at a very large rate. Thus it has been measured that 1 gramme of radium emits heat-energy at the rate of 100 calories or 140,000 foot-pounds per hour.

This energy is not available for mechanical purposes, but it is not unlikely that in the future it may be used for purposes of illumination. Possibly a part of the heat of the Sun and the Earth may be due to the energy emitted by disintegrating atoms.

EXERCISES X. (*continued*).

9. What work is done against gravity in drawing a mass of 1 ton up an incline 100 yds. long and 5 yds. high?

10. What amount of work is done by an engine in increasing the velocity of a truck weighing 1 ton from 2 ft. per sec. to 5 ft. per sec.?

11. Find the work done in carrying up the materials of a square tower 100 ft. high, if each foot of the height of the tower contains 3000 lbs. mass of bricks, &c.

12. A stone of 2 lbs. mass is moving horizontally with a velocity of 400 ft. per sec., and is brought to rest by penetrating 4 ft. into a mound of sand. Find the average resistance of the sand.

13. When is the energy of a pendulum bob (i.) wholly potential, (ii.) wholly kinetic?

14. When an arrow is on the point of release the bow possesses potential energy. What is the difference between this case of potential energy and that of a body raised above the level of the Earth?

15. When the tire of a bicycle is being inflated, the pump becomes very hot. What is the source of this heat?

16. Why does the barrel of a gun become hot after several shots have been projected?

17. Describe how the energy of water which falls from a height into a pond without outlet is transformed.

18. Describe the transformation of energy when a spark is produced by means of a flint and steel.

19. Describe the transformations through which energy passes as clouds are formed and as rain returns from them to the Earth.

20. Describe the passage of energy from the coal to the passengers and the rails when a train is in motion. Where else has energy gone?

21. In a shipbuilding yard a machine pierces holes in iron plates by punching out circular fragments. Before beginning work the machinery and iron plates are quite cold. After the operation the circular fragments are too hot to hold in the hand. Why is this? How could you ascertain experimentally the amount of heat gained by one of the fragments during the process?

Summary.—Chapter X.

1. When an agent produces motion in a body it is said to do work; and the work is measured by the product of the force employed and the distance traversed parallel to the line of action of the force, *i.e.*

$$W = F.s. \qquad (\S\S\ 94, 95.)$$

2. The absolute units of work are the foot-poundal and the erg. (§ 96.)

3. The practical units of work are the foot-pound and the kilogrammemetre.

1 foot-pound = 32 foot-poundals. (§ 97.)

1 kilogrammetre = 98,100,000 ergs. (§ 97.)

4. A horse-power = 550 ft.-lbs. of work per second
= 33,000 ft.-lbs. of work per minute
= 746 watts. (§ 99.)

5. Energy is capacity for doing work. Kinetic energy is energy due to motion. Potential energy is energy due to position. (§§ 100, 101, 104.)

6. The kinetic energy of a mass in lbs. moving with a velocity of u ft. per sec. is $\frac{1}{2}mu^2$ foot-poundals

$$= \frac{1}{2}\frac{mu^2}{g} \text{ ft.-lbs., where } g = 32. \qquad (\S\ 102.)$$

If m is in gms. and u in cms. per sec., the

$$\text{K.E.} = \frac{1}{2}mu^2 \text{ ergs} - \text{dyne cm}$$

$$= \frac{1}{2}\frac{mu^2}{g} \text{ gm.-cms., where } g = 981. \qquad (\S\ 102.)$$

7. The *Equation of Energy*.—If a force F acts through a distance s upon a mass m and increases its velocity from u to v,

work done by force = increase of K.E. of body,

i.e. $$Fs = \frac{1}{2}mv^2 - \frac{1}{2}mu^2. \qquad (\S\ 103.)$$

$\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

8. For a freely falling body at any instant,

$$\text{P.E.} + \text{K.E.} = \text{a constant},$$

the constant being equal to the P.E. of the body before its fall or to the K.E. of the body when it reaches the ground. (§ 105.)

9. Energy is transferable from one body to another. (§ 106.)

10. There are many different forms of energy. Any one form, under suitable conditions, may be transmuted into other forms, and, when all the forms are taken into account, there is no loss or gain of energy. This is known as the *Conservation of Energy*, a theorem of the greatest importance. (§§ 107–112.)

11. The theorem of the Conservation of Energy gives us the *Principle of Work*, by which we may find the ratio between the effort and resistance in any one of the simple machines, assuming that no energy is lost by friction, &c. (§ 113.)

12. Energy may be stored in many different ways, and transmitted in many different forms. (§ 114.)

EXERCISES X. (*concluded*).

22. How many absolute units of work (ft.-poundals or ergs) are done by
(i.) A man of mass 10 stones in walking up stairs to a height of 24 ft.;
(ii.) A force of 20 gms. wt. acting through a distance of $1\frac{1}{2}$ metres?

23. If a man can work at the rate of 210,000 ft.-lbs. per hour, how long would it take him to raise a weight of 10 tons through 150 ft., supposing him to be provided with a suitable machine?

24. If the mass of a body is 15 lbs. and its velocity 12 ft. per sec., how many foot-poundals of work can it do against a resistance in virtue of its mass and velocity?

25. A body whose mass is 10 lbs. is capable of doing 605 ft.-poundals of work in virtue of its mass and velocity. At the rate of how many feet per sec. is it moving?

26. A body whose mass is 6 lbs. is moving at the rate of 8 ft. per sec. How many foot-poundals of work can it do against a resistance, in virtue of its mass and velocity? If it did 117 ft.-poundals of work against a resistance, what would then be its velocity?

27. AB is a rod 20 ft. long, that can turn freely round the end A; at B a force of 35 lbs.-wt. is applied at right angles to AB; the rod is allowed to turn six times. How many foot-pounds of work are done by the force?

28. A body weighing 10 lbs. is placed on a horizontal plane and is made to slide over a distance of 50 ft. by a force of 4 lbs. What number of units of work is done by the force? At the instant the 50 ft. have been described, what is there in the state of the body to show that work has been done on the body?

29. Find the kinetic energy of the following masses moving with the stated velocities :—

(i.) 4 lbs., 3 ft. per sec. ; (iii.) 2000 gms., 50 cms. per sec.

(ii.) 10 stones, 3 miles per hour ; (iv.) 300 tons, 60 miles per hour.

30. A body of mass 100 lbs. is observed to be moving at the rate of 20 ft. per sec. Assuming that it began to move from a state of rest, and that its motion was impeded by no resistance, how many units of work must have been done on it by the force that gave it the velocity? Give the answer (*a*) in foot-poundals, (*b*) in foot-pounds.

31. A body whose mass is 10 lbs. is moving at the rate of 50 ft. per sec. What is the numerical value of its kinetic energy at that instant? If from that instant it moves against a constant resistance equal to one-twentieth of its weight, how far does it go before being brought to rest? ($g = 32$.)

32. Through what distance must a force of 1 lb.-wt. act upon a mass of 48 lbs. to increase its velocity from 24 to 36 ft. per sec. ?

33. A body whose mass is 10 lbs. is carried up to the top of a house 30 ft. high. By how many foot-poundals has the change of position increased its potential energy? If it is allowed to fall, what number of foot-poundals of kinetic energy will it have when it reaches the ground?

34. How many ft.-lbs. of work are done in raising the soil of a pit measuring 12 ft. broad, 8 ft. long, and 10 ft. deep, if each cubic foot of earth weighs 84 lbs. ?

35. A body whose mass is 20 lbs. moves in a straight line against a constant resistance. At a certain point it is moving at the rate of 18 ft. per sec. After moving over 50 ft. its velocity is reduced to 10 ft. per sec. What part of its kinetic energy has it lost? What is the numerical value of the resistance in poundals?

36. The mass of a particle is 10 lbs. At a certain instant it is moving at the rate of 24 ft. per sec. ; it moves against a constant resistance of 4 poundals. What distance would it describe from that instant before coming to rest?

37. A particle moving from rest is acted on through 250 ft. by a force of 9 poundals. Find its kinetic energy ; and, its mass being 5 lbs., find its velocity.

38. A particle whose mass is 12 lbs. has its velocity changed from 5 ft. per sec. to 11 ft. per sec. What number of foot-poundals of work has been done by the force to which the change is due?

39. A body whose mass is 5 lbs. drops through a distance of 100 ft. into some soft mud. It is brought to rest in 10 ft. What is the average resistance of the mud?

40. In the case of the simple lever show that the work done by the effort is equal to the work done against the resistance.

ANSWERS.

SECTION I.—MEASUREMENT AND MATTER.

Exercises I. (*Page* 14.)

8. 99·41 cms. **9.** 4·796 kilometres. **10.** 24850 miles.
11. 29·92 inches. **12.** 981·2 cms. per sec. **13.** 33·8 kilometres.
14. $\frac{1}{4\,3\,1\,0\,0}$ inch.

Exercises II. (*Page* 19.)

4. 4·24 inches.

Exercises III. (*Page* 22.)

1. 15·71 inches, 31·42 cms., $\pi = 3{\cdot}14$.

Exercises IV. (*Page* 36.)

2. Perpendiculars 6·93, 8·66 cms., Area 69·3 sq. cms.
3. 6 sq. cms. **4.** 9·8 sq. cms. **5.** 0·785.
8. 1 acre 1 rood $4\frac{1}{2}$ sq. poles. **11.** 9·94.
12. 462 sq. ins. **13.** ·001. **14.** $8\frac{8}{9}$ acres.

Exercises V. (*Page* 42.)

1. 16·39. **2.** 61·03.

Exercises VI. (*Page* 45.)

12. 20·412. **13.** 62·8. **14.** 1,000. **15.** 10·6. **16.** 27·8.

Exercises VII. (*Page* 49.)

1. 345·6 lbs. per cub. ft. **2.** $\frac{1}{2}$ cub. ft. **3.** 8·87.
4. 4,000 lbs. **5.** 5 : 12. **6.** 1·728, 1·327. **7.** 4·2.
8. 1·2. **9.** $46\frac{121}{140}$ tons.

EXERCISES VIII. (*Page* 53.)

8. $133\frac{1}{3}$. **9.** 2·5. **10.** 220·2 kilogs. **11.** 4.
12. 154 ft. **13.** ·786. **14.** 1·39. **15.** ·72.

EXERCISES IX. (*Page* 61.)

9. 62·5 lbs. **10.** 75 lbs. **11.** Thread vertical ; $1\frac{2}{7}$ lbs.
12. 116 oz. **13.** 2·56. **14.** $108\frac{1}{2}$ oz.
15. 8 ; 34·56. **16.** 450 grains. **17.** 9·5 gms.
18. ·946. **19.** ·848. **20.** 1·1. **21.** ·96.

EXERCISES X. (*Page* 80.)

1. 864 : 25. **2.** 64 lbs. **3.** 192 lbs. **4.** $13\frac{8}{9}$.
5. 15 lbs. per sq. in. **6.** 33·4 ft. **7.** 1·26.
8. 4 ins. ; 36 ins. **9.** Enough to fill 12 ins. of the tube.
10. 1·122 kilog. **11.** 1·115.

EXERCISES XI. (*Page* 83.)

1. $y = 2x$. **2.** $x = 3y$. **3.** $x^2 + y^2 = 9$.
5. A straight line whose equation is $x - y = 2$.
6. $x = 1{\cdot}55$, $y = 1{\cdot}4$.

EXERCISES XII. (*Page* 96.)

3. The principle of the lever.
6. (i.) 1 ft. from 3 lbs. ; (ii.) $12\frac{1}{2}$ ins. from 7 lbs.

ANSWERS.

SECTION II.—MECHANICS.

Exercises I. (*Page* 11.)

7. A line 2 ins. long pointing N.W. or S.E.

8. (i.) 21; (ii.) $\frac{21}{2}\sqrt{3}$; (iii.) $\frac{21}{2}\sqrt{3}$; (iv.) $10\frac{1}{2}$ in.-oz. units.

9. $\frac{5}{7}$ units of length.

10. (*a*) 300, (*b*) 750; (*b*) in that of the 50 lbs. force.

11. 40, 28, 20. **12.** 4, 6, $7\frac{1}{5}$.

Exercises II. (*Pages* 20, 22, 30, 31.)

1. (i.) 3 units, 1 ft. from greater, (ii.) 8 units, 9 ins. from greater; (iii.) $\frac{1}{2}$ lb., $\frac{1}{2}$ yd. from greater; (iv.) 60 gms., 13·8 ins. from greater.

2. (i.) 1 unit, 3 ft.; (ii.) 2 units, 3 ft.; (iii.) $\frac{5}{14}$ lb., $\frac{7}{10}$ yd.; (iv.) 14 gms., $1\frac{9}{14}$ yds.

3. 2 units and 10 units like, and 12 units at $\frac{1}{6}$ of distance from the 10 units force to the two units force.

4. 6 ft. $1\frac{1}{3}$ ins. **6.** $4\frac{4}{19}$ ft. from heavier; 19 lbs.

7. 7, $3\frac{5}{7}$ from *A*; 2, $4\frac{1}{2}$ from *A*; -3, $8\frac{2}{3}$ from *A*.

8. (i.) $\frac{3}{4}$ ins.; (ii.) $6\frac{2}{3}$ ft.; (iii.) $3\frac{1}{3}$ cms. from heavier.

9. 6000 miles from centre of the Earth.

14. $\frac{1}{10}$ of diagonal from centre.

15. 12 units, $\frac{5}{12}$ of distance from greater.

16. 1 unit at distance $3a$ from 4 units force, where a is the distance between the forces.

17. 12, $2\frac{11}{12}$ ft. **18.** $1\frac{1}{2}$, $\frac{1}{2}$. **19.** $2\frac{1}{2}$, $\frac{1}{2}$.

20. 10 ins. from the greater; 5 ft. from the greater.

21. 10 ft. from the larger. **22.** $6\frac{2}{3}$, $3\frac{1}{3}$ lbs.

23. *R* varies inversely as the distance between the hand and the shoulder.

25. 8 lbs. **26.** 4 ft. from the stronger.

27. $33\frac{1}{3}$ cms. from the fulcrum. **28.** $\frac{1}{6}$ of diagonal from centre.

29. At a point on the former distant $\frac{3}{10}$ of its length from the junction.

30. $\frac{1}{7}$ of diag. from centre. **31.** $8\frac{17}{43}$ ins. from end of longer cylinder.

33. At point of trisection of median nearer the particle.

35. The vertical through the suspended corner will cut the line joining the centre to the 4 oz. at a distance from the centre equal to $\frac{1}{3}$ of the length of the line.

Exercises III. (*Pages* 36, 39, 40.)

1. About 15·9 lbs. wt. **2.** (i.) 50 lbs., 37°. (ii.) 65 lbs., $22\frac{1}{2}$°.
3. 11·4 kilogs. (iii.) 5·8 tons, 31°. (iv.) 13·5 tons, 42
4. 5, $53\frac{1}{8}$°. **5.** The angles between them must be 55°, 138°, 167°.
6. $141\frac{1}{2}$, 125, $93\frac{1}{2}$.
7. 10 is greater than the sum of 5 and 4. **8.** $1 : \sqrt{3} : 2$.
9. 28·3 N. and 28·3 E. **10.** 86·6 S. and 50 W.
13. Angle between equal forces = 67°.
15. $52\frac{1}{2}$°. **16.** 151°. **17.** 27, 33 units.
18. Horizontal string, $8\sqrt{3}$ lbs.; inclined string, $16\sqrt{3}$ lbs.

Exercises IV. (*Page* 49.)

1. 250,433 gms. **2.** $\frac{5}{3}$, $\frac{5}{4}$. **3.** 1 in 8.
4. 65 lbs. **5.** 84 lbs. **6.** Along the plane.

Exercises V. (*Page* 56.)

1. Equal. **2.** 4 : 5. **3.** g is different at the two places.
4. 621 cms. **5.** $8\frac{8}{9}$ ft. **6.** $l_A : l_B = 4 : 9$.

Exercises VII. (*Page* 69.)

1. $\frac{5}{7}$, $29\frac{1}{3}$, $\frac{1}{12}$, x/n, $x/1200n$. **2.** 3 : 7. **3.** (i.) 152·4; (ii.) 1430.
4. 1530. **5.** $22ny/15x$ hours. **6.** 40 miles per hour.
7. 80 ft. per sec. **8.** 11, 5, −3 miles per hour. **9.** 88 ft. per sec.
10. 3 : 8. **11.** 30·48 cms. per sec. **12.** ·0328 ft. per sec.
13. 100 secs. **14.** (i.) 120; (ii.) $4\frac{1}{11}$; (iii.) 183, nearly.
15. 9 miles per hour up-stream. **16.** 52·8 ft. per sec.
17. $2\frac{1}{2}$, 3, $\frac{1}{2}$ miles. **18.** 13, 16·5, 14·7, 9·2, 7·3.
19. 37·4 miles per hour. **20.** 3·32 miles per hour.
21. 5 f.s.; $\sqrt{29}$ or 5·4 f.s. **22.** 11·5, 23·1 f.s.
24. 120°. **25.** 35·3 miles per hour.

Exercises VIII. (*Pages* 77, 82.)

1. 150 ft., 48 f.s. **2.** 63·1; 5·73 f.s. **4.** $29\frac{1}{3}$ secs.
5. 1920, 38400. **6.** 24000. **7.** 1600 ft., 320 f.s.
8. 8 f.s., 1 ft. **9.** $4\frac{1}{4}$ secs. **10.** $5\frac{5}{16}$ secs.; no resistance.
11. 864 ft., 464 f.s. **12.** 121 ft.

EXERCISES IX. (*Pages* 91, 96.)

1. 24. **2.** 9 : 10, 5 : 8. **3.** 6 f.s.s.; uniform.
4. (i.) 7 : 8, (ii.) 7 : 8. **5.** 60, 5 : 32. **6.** 2 : 7.
7. 56; 300,000. **8.** Equal.
9. 160; 7850; 6; 1792; 6,870,000. **10.** 3.
11. $\frac{1}{4}$; **12.** 3 : 20. **13.** 140 poundals. **14.** 1 poundal.
15. No; the indications of the balance vary with intensity of gravity.
16. 768 poundems. **19.** 32 f.s.s. **20.** 1st Law; 3rd Law.
22. 75 : 16; $7\frac{1}{2}$ f.s.s. **23.** 50 poundals.
24. $1\frac{3}{5}$ f.s.s; 21500 poundems. **25.** 161 poundals; 5.

EXERCISES X. (*Pages* 102, 115, 117.)

2. (i.) 600 ft.-lbs.; (ii.) 3,360 ft.-lbs.; (iii.) ·03 kilogrammetre.
3. 12,000; 5,000. **4.** In ft.-lbs.
5. $4{\cdot}18\times10^7$; ·424. **6.** 2×10^5. **7.** 1 hour 40 mins.
8. 240. **9.** 33,600 ft.-lbs. **10.** 25,320 ft.-poundals.
11. $1{\cdot}5\times10^7$ ft.-lbs. **12.** 40,000 poundals.
21. By a specific heat process (See Heat, § 21). **22.** 107500; 2943000.
23. 16 hours. **24.** 1080. **25.** 11.
26. 192; 5 f.s. **27.** 26400. **28.** 200 ft. lbs.; kinetic energy.
29. (i.) 18; (ii.) 1355; (iii.) 2500000; (iv.) 2600000000 ft.-poundals.
30. (*a*) 20000; (*b*) 625. **31.** 12500 ft.-poundals; 781 ft. **32.** 540 ft.
33. 9600; 9600. **34.** 403000. **35.** 2240 ft.-poundals, 44·8.
36. 720 ft. **37.** 2250 ft.-poundals; 30 f.s.
38. 576 ft.-poundals. **39.** 55 lbs.-wt.

PRINTED AT THE BURLINGTON PRESS, CAMBRIDGE.

A complete Catalogue of Text-Books published by the University Tutorial Press, and separate Sectional Catalogues in English Language and Literature, French, Mathematics, and Science, may be had on application to the Publisher.

SELECTED TEXT-BOOKS IN SCIENCE

PUBLISHED BY THE

University Tutorial Press Ld.

25 High St., New Oxford St., W.C.

Chemistry.

The Tutorial Chemistry. By G. H. Bailey, D.Sc., Ph.D. Edited by Wm. Briggs, LL.D., M.A., B.Sc., F.C.S.

Part I. Non-Metals. *Fourth Edition.* **3s. 6d.**

Part II. Metals and Physical Chemistry. *Second Edition.* **4s. 6d.**

Qualitative Analysis. By Wm. Briggs, LL.D., M.A., B.Sc., F.C.S., and R. W. Stewart, D.Sc. **3s.**

Quantitative Analysis. By Wm. Briggs, LL.D., M.A., B.Sc., F.C.S., and H. W. Bausor, M.A. **2s.**

Senior Volumetric Analysis. By H. W. Bausor, M.A. **1s. 6d.**

Bailey's Elementary Chemistry. By G. H. Bailey, D.Sc., Ph.D. Edited by Wm. Briggs, LL.D., M.A., B.Sc. *Fourth Edition.* **4s. 6d.**

An Introductory Course of Chemistry. By H. W. Bausor, M.A. **2s. 6d.**

Chemical Calculations. By H. W. Bausor, M.A. **2s.**

Chemical Calculations (Advanced Course). By H. W. Bausor, M.A. **1s.**

Chemistry, Preliminary. By H. W. Bausor, M.A. **1s. 6d.**

Chemistry—*continued.*

Chemistry, First Stage Inorganic (Theoretical). By G. H. Bailey, D.Sc. *Fourth Edition.* **2s.**

Chemistry, First Stage Inorganic (Practical). *Revised Edition.* By H. W. Bausor, M.A. **1s.**

Chemistry, Second Stage Inorganic (Theoretical). By G. H. Bailey, D.Sc., Ph.D. Edited by Dr. Wm. Briggs. *Fourth Edition.* Revised by H. W. Bausor, M.A. **4s. 6d.**

Chemistry, Second Stage Inorganic (Practical). By Dr. Wm. Briggs and Dr. R. W. Stewart. *Third Edition.* **2s. 6d.**

Qualitative Determination of Organic Compounds. By J. W. Shepherd, B.Sc. **6s. 6d.**

The Elements of Organic Chemistry. By E. I. Lewis, B.A., B.Sc. **2s. 6d.**

Introduction to the Carbon Compounds. By R. H. Adie, M.A., B.Sc. **2s. 6d.**

Physics.

The Tutorial Physics. By R. W. Stewart, D.Sc., E. Catchpool, B.Sc., C. J. L. Wagstaff, M.A., W. R. Bower, B.Sc., A.R.C.S., and J. Satterly, D.Sc., M.A.

A series in Six Volumes suitable for University Classes and for the highest forms of secondary schools.

I. Sound, Text-Book of. By E. Catchpool, B.Sc. *Fifth Edition.* **4s. 6d.**

II. Heat, Higher Text-Book of. By R. W. Stewart, D.Sc. **6s. 6d.**

III. Light, Text-Book of. By R. W. Stewart, D.Sc. **4s. 6d.**

IV. Magnetism and Electricity, Higher Text-Book of. By R. W. Stewart, D.Sc. *Second Edition.* **6s. 6d.**

V. Properties of Matter. By C. J. L. Wagstaff, M.A. **3s. 6d.**

VI. Practical Physics. By W. R. Bower, B.Sc., A.R.C.S., and J. Satterly, D.Sc., M.A. **4s. 6d.**

Heat, Theoretical and Practical, Text-Book of. By R. W. Stewart, D.Sc., and John Satterly, D.Sc., M.A. **4s. 6d.**

Magnetism and Electricity, The School. By R. H. Jude, M.A., D.Sc. **3s. 6d.**

Physics—*continued.*

Magnetism and Electricity, First Stage. By R. H. Jude, M.A., D.Sc. *Third Edition.* **2s.**

Magnetism and Electricity, Second Stage. By R. W. Stewart, D.Sc. *Second Edition.* **3s. 6d.**

Mathematical Physics, Vol. I., Magnetism and Electricity. By C. W. C. Barlow, M.A., B.Sc. **4s. 6d.**

Junior Experimental Science. By W. M. Hooton, M.A., M.Sc., F.I.C. *Second Edition.* **2s. 6d.**

Experimental Science, Preliminary Course of. By W. M. Hooton, M.A., M.Sc. **1s. 6d.**

Elementary Science, General. Edited by Wm. Briggs, LL.D., M.A., B.Sc., F.R.A.S. **3s. 6d.**

Elements of Physical Science. Specially prepared to meet the requirements of the Matriculation Examination of the Cape University. Edited by William Briggs, LL.D., M.A., B.Sc., F.C.S. *Second Edition.* **3s.**

Mechanics and Physics, An Introductory Course of. By W. M. Hooton, M.A., M.Sc., and A. Mathias. **1s. 6d.**

Biology.

Text-Book of Botany. By J. M. Lowson, M.A., B.Sc., F.L.S. *Fifth Edition, Revised and Enlarged.* **6s. 6d.**

Text-Book of Botany. (Indian Edition.) By J. M. Lowson, M.A., B.Sc., F.L.S., and M. Willis. **6s. 6d.**

Practical Botany. By F. Cavers, D.Sc., F.L.S. **4s. 6d.**

Ewart's Elementary Botany. By A. J. Ewart, D.Sc. **3s. 6d.**

The Senior Botany. By F. Cavers, D.Sc., F.L.S. **4s. 6d.**

First Course in Plant and Animal Biology. By W. S. Furneaux. **2s.**

Plant Biology. By F. Cavers, D.Sc., F.L.S. **3s. 6d.**

Life Histories of Common Plants. By F. Cavers, D.Sc., F.L.S. **3s.**

Biology—*continued.*

Botany, First Stage. By A. J. Ewart, D.Sc., F.L.S. *Second Edition.* **2s.**

Botany, Second Stage. By J. M. Lowson, M.A., B.Sc. *Fourth Edition, Revised and Enlarged.* **4s. 6d.**

Zoology, Text-Book of. By H. G. Wells, B.Sc., and A. M. Davies, D.Sc. **6s. 6d.**

Physiology, First Stage Human. By G. N. Meachen, M.D., B.S. Lond., M.R.C.P., M.R.C.S. **2s.**

Applied Sciences, etc.

Building Construction, First Stage. By Brysson Cunningham, B.E., A.M.I.C.E. *Second Edition.* **2s. 6d.**

Electrical Engineering. By W. T. Maccall, M.Sc., A.M.I.E.E. [*In preparation.*

Electricity, Technical. By Professor H. T. Davidge, B.Sc., Professor of Engineering at the Ordnance College, Woolwich, and R. W. Hutchinson, M.Sc., Principal of the Municipal Technical School, Smethwick. *Third Edition.* **5s. 6d.**

Heat Engines, First Stage. By J. W. Hayward, M.Sc. **2s. 6d.**

Geometry, First Stage, Practical Plane and Solid. By G. F. Burn, A.M.I.Mech.E. *Third Edition.* **2s.**

Hygiene, First Stage. By R. A. Lyster, M.D., B.Sc., D.P.H. *Sixth Edition.* **2s. 6d.**

Hygiene, Second Stage. By A. E. Ikin, B.Sc., LL.B., and R. A. Lyster, M.D., D.P.H. *Third Edition.* **3s. 6d.**

Modern Navigation (Navigation and Nautical Astronomy). By William Hall, B.A., R.N. **7s. 6d.**

Science French Course. A Course in French for Science Students. By C. W. P. Moffatt, M.A., M.B., B.C. **3s. 6d.**

Science German Course. A Course in German for Science Students. By C. W. P. Moffatt, M.A., M.B., B.C. *Second Edition.* **3s. 6d.**

stretching is prep to the
Love producing it

Physics
S

Satterly, John
Measurement and m echanics.

345093

DATE

NAME OF BORROWER

University of Toronto
Library

DO NOT
REMOVE
THE
CARD
FROM
THIS
POCKET

Acme Library Card Pocket
LOWE-MARTIN CO. LIMITED